Actuarial Fairness in Redistributive Pension Systems*

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Abstract

Most countries in the OECD have a Delayed Retirement Credit (DRC) in their public pension systems, defined as the percentage increase in the annuity value of pension benefits from delaying retirement. If the DRC is actuarially fair, then it is just large enough to compensate the individual for the extra taxes paid and the foregone pension benefits from delaying retirement (Sheshinski (2008a)). In this paper we analytically derive the actuarially fair DRC when there is wage heterogeneity and the pension system is redistributive. We prove under general conditions that the actuarially fair DRC must be increasing in earnings. The more redistributive the benefit-earning formula in the pension system, the greater is the need for an earnings-based DRC. This result stands in stark contrast to the fact that the DRC is unrelated to earnings in all OECD countries.

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1 Introduction

Most countries in the OECD have a Delayed Retirement Credit (DRC) in their public pension systems, defined as the percentage increase in the annuity value of pension benefits from delaying retirement. For example, the US social security system pays an annuity to those who retire at age 70 that is 76% larger than the annuity collected by those who retire at age 62. While the exact size of the DRC differs across countries, its basic purpose is to make the pension system actuarially fair to prevent premature departure from the labor force. Fairness is achieved if the DRC is just large enough to compensate the individual for the extra taxes paid and the foregone pension benefits from delaying retirement (Sheshinski (2008a)).

In this paper we analytically derive the actuarially fair DRC when there is wage heterogeneity in the economy. We consider pension systems that range from Bismarckian to Beveridgean.[1] We prove under general conditions that the DRC must be increasing in earnings in order to achieve actuarial fairness, as long as the system is not Bismarckian. This result stands in stark contrast to the fact that the DRC is unrelated to earnings in all OECD countries: the rich and poor alike earn the same percentage increase in the annuity value of benefits for delaying retirement. A DRC that is unrelated to earnings would be actuarially fair only in the special case where the pension system is Bismarckian.

We also show that our theoretical results are quantitatively important. For example, calibrating to the US benefit-earning rule (to capture the degree of redistribution in the US system), we find that the DRC for the rich would need to be as much as 30% higher than the DRC for the poor to achieve actuarial fairness. And the fair DRC for the rich in a purely Beveridgean system would need to be as much as 69% higher than the DRC of the poor.

Our paper is closely related to two strands of literature. The first is due to Sheshinski (2008a). He develops an abstract, elegant model that can be used to understand the conditions under which a DRC is desirable in the first place. The main mechanism at play in his paper is heterogeneity in labor disutility. The second strand of literature is quite large and it uses cross-country data to show that the DRC is a key determinant of labor force participation at older ages (e.g., Gruber and Wise (1997, 1998)).[2] Building on the idea that the DRC is both desirable and empirically important in predicting labor force participation in old age, we show how to design an actuarially fair DRC in a realistic world with wage heterogeneity and a redistributive benefit-earning rule.

2 Basic Notation and a Review of Standard Life-Cycle Consumption Theory

Note that the DRC is designed for those who are prone to late retirement in the absence of a public pension system. For these individuals, a system which provides no rewards would

[1] A Bismarckian pension system is one in which there is no redistribution of pension wealth, meaning that everyone has the same replacement rate. A Beveridgean system is at the other end of the spectrum—everyone gets the same benefits regardless of contributions. The US system is between the two extremes but is closer to Bismarckian than to Beveridgean.

[2] Prescott (2004) opened up a similar literature, which seeks to understand the effects of tax-and-transfer programs on hours of work. Also see Ortiz and Rogerson (2009) and Ortiz (2009).

create a temptation to collect benefits as soon as possible, and since early collection before the normal retirement age is not possible while working without a massive tax penalty (in the US), the individual would be tempted to leave work early to collect benefits. If the rewards for delayed retirement (or equivalently, the penalties for early retirement) are structured to be actuarially fair, meaning the individual could not increase the net present value of pension benefits and taxes by retiring early and collecting benefits, then the early-retirement distortion is removed. We now formalize these ideas in a standard life-cycle model.

Time (age) is continuous and is indexed by t. At $t = 0$ the individual enters the workforce, at $t = T$ he retires endogenously, and then he passes away no later than $t = \bar{T}$ (the maximum lifespan). The probability of surviving to age t from the perspective of age 0 is $S(t)$. The probability of passing away at age t, from the perspective of time zero, is $-dS(t)/dt$. Let $S(0) = 1$ and $S(\bar{T}) = 0$.

Let $c(t)$ be the flow of consumption at age t. For simplicity and without loss of generality, we have two groups of consumers, rich (denoted by $+$) and poor (denoted by $-$). We can instead have a continuum of earners and the results are the same. The rich earn wage income at rate w^+ and the poor earn wage income at rate w^-. All workers pay pension taxes at rate θ. The respective groups receive pension benefits (a constant annuity) during the retirement period at rate $b^+(T)$ and $b^-(T)$, which depend on the date of retirement and will be explained below.

We consider a world that abstracts from capital or savings accounts and instead there are competitive annuities (insurance contracts) that are used to plan for retirement. This is not a critical assumption as *all* of our theoretical results will continue to hold if we close annuity markets and instead model savings accounts.[3] Our treatment of annuities follows Sheshinski (2008b). We treat $a(t)$ as the quantity of annuities held by the individual. We assume annuities can be bought and sold at unit price (essentially we are assuming a fully developed residual annuity market where annuities can be resold, which for convenience we can think of in terms of selling back to the originator at unit price). All individuals start and stop the life cycle with no annuities, $a(0) = a(\bar{T}) = 0$. The holder of an annuity collects a flow of returns for as long as he lives. The return $r(t)$ depends on his age. There are no bequests because deceased individuals surrender their annuities as part of the contract, and these annuities are then given to survivors of the same age as the deceased. The annuity market is competitive—zero profits for the administrator of the annuities—because all surrendered annuities are distributed to survivors leaving nothing for the administrator of the contract and annuities can be sold back to the administrator at unit price. Therefore, perfect competition implies[4]

[3] Essentially, there are two fundamental differences between this model and one with capital: (i) here the rate of return on investment grows with age whereas in a standard model with capital it is constant, and (ii) here the assets of the deceased are given to individuals who share the same age as the deceased whereas in models without annuities it is often assumed that accidental bequests are spread evenly across the surviving population.

[4] Readers less familiar with annuities may find this footnote helpful, which proves that equation (1) is the result of zero profit. Suppose at each moment a new cohort is born. Each new cohort contains a size N mass of infinitely divisible individuals (which can be normalize to 1). The share of poor in the cohort is μ and the share of rich is $(1 - \mu)$. Because we have assumed a continuum of agents in every cohort, the survival probability $S(t)$, which applies to both the rich and poor, can be interpreted as the actual percentage of a given cohort that is alive at age t. And $-dS(t)/dt$ is the fraction of a cohort who die at age t. Therefore $-\mu N\,(dS(t)/dt)\,a^-(t)$ is the quantity of annuities surrendered by the poor who die at age t

$$r^{-}(t) = r^{+}(t) = r(t) = \frac{-dS(t)/dt}{S(t)} = -\frac{d\ln S(t)}{dt}. \tag{1}$$

The optimal consumption path, for either the rich or the poor, would maximize expected utility. We assume no impatience, other than discounting for the sake of mortality. This keeps the notation simple but does not in any way affect our results since pure time preference does not enter the budget constraint. The optimal consumption path and the optimal date of retirement are the solutions to the following control problem

$$\max_{c(t),T} : \int_0^{\bar{T}} S(t)u[c(t)]dt - \phi(T), \tag{2}$$

where u is the period utility of consumption with $u_c > 0$ and $u_{cc} < 0$, and $\phi(T)$ is disutility of work ($\phi'(T) > 0$). We assume retirement is a binary choice to abstract from the intensive leisure margin for simplicity.

The constraints of the control problem are

$$\frac{da(t)}{dt} = r(t)a(t) + y(t) - c(t), \tag{3}$$

$$y(t) = (1-\theta)w, \text{ for } t \in [0, T], \tag{4}$$

$$y(t) = b(T), \text{ for } t \in \left[T, \bar{T}\right], \tag{5}$$

$$a(0) = 0, \tag{6}$$

$$a(\bar{T}) = 0. \tag{7}$$

In (3) we see the change in annuities is due to collections from the deceased, $r(t)a(t)$, and new purchases (or sales), $y(t) - c(t)$.

This is a two-stage fixed endpoint control problem. The switch from the first to the second stage is endogenous. Note that the date of retirement T enters the problem in three ways: (i) the disutility of work is directly related to the length of the worklife, (ii) the timing of the switch in the differential equation, and (iii) the annuity value of pension benefits (which accounts for the DRC). To solve, we can first note that, for any retirement date, the individual would choose consumption optimally. Because of the zero profit condition (1), the optimal consumption path is constant all across the life cycle for any period utility function and any survival function. Using the Maximum Principle and treating T as fixed we obtain

$$c = \left[\int_0^{\bar{T}} y(t)S(t)dt\right] \times \left[\int_0^{\bar{T}} S(t)dt\right]^{-1}. \tag{8}$$

and $-(1-\mu)N\,(dS(t)/dt)\,a^{+}(t)$ is the quantity surrendered by the rich. The mass of poor and rich annuity holders who survive to age t is $\mu NS(t)$ and $(1-\mu)NS(t)$. Due to zero profits in the annuity market, and assuming the annuities of the deceased are given to survivors in proportion to the holdings of the two groups, we can divide the quantity of annuities surrendered by the surviving population to get annuities received, per-survivor. This is akin to interest income from a savings account: $[-dS(t)/dt]a^{-}(t)/S(t)$ for the poor and $[-dS(t)/dt]a^{+}(t)/S(t)$ for the rich. Hence, the zero profit condition can be written as (1).

Notice that the first term on the right of (8) is expected income and the second term can be shown to be the inverse of life expectancy, which is $\int_0^{\bar{T}} t[-dS(t)/dt]dt$ (see Sheshinski (2008b)). Thus, for a given retirement date, it is optimal to set consumption equal to expected income per year of life expectancy.

Now, the optimal retirement age is

$$T = \arg\max \left[\int_0^{\bar{T}} S(t)u \left(\left[\int_0^T wS(t)dt + V(T) \right] \times \left[\int_0^{\bar{T}} S(t)dt \right]^{-1} \right) dt - \phi(T) \right], \quad (9)$$

where

$$V(T) \equiv \int_T^{\bar{T}} S(t)b(T)dt - \int_0^T S(t)\theta w dt \quad (10)$$

is the expected net present value of the public pension from the individual's perspective. The pension will distort the retirement choice if and only if the net present value is indeed a function of the retirement choice. Therefore, the goal is to design the DRC in a way that makes V invariant to T.

3 Main Analytical Results

The issue that interests us is how to design a DRC so that extra years of work, and hence extra taxes paid and a shorter benefit annuity received, are properly rewarded. If there is a DRC in the public pension system, then benefits depend positively on the age at which the individual retires and benefits typically depend on the wage earnings of the individual. Thus,

$$\frac{db^i(T)}{dT} > 0, \text{ for } i \in \{-,+\}, \text{ and } b^-(T) \leq b^+(T) \leq b^-(T)\frac{w^+}{w^-}, \quad (11)$$

for all $T \in [T_{\min}, T_{\max}]$, where the minimum eligibility age is $T_{\min}$ and $T_{\max}$ is the age after which no more rewards are possible. If $b^-(T) = b^+(T)$, the pension system is Beveridgean. If $b^+(T) = b^-(T)w^+/w^-$, the pension system is Bismarckian because benefits are proportional to wages (contributions) and hence everyone experiences the same internal rate of return. The US program is somewhere between (we will be more specific in the next section).

The net present value V of pension benefits and taxes is a function of income type (i) and retirement age (T)

$$V^i(T) = \int_T^{\bar{T}} S(t)b^i(T)dt - \int_0^T S(t)\theta w^i dt, \text{ for } i \in \{-,+\}. \quad (12)$$

We use the term actuarial fairness to mean that the date of retirement does not affect the net present value of the public pension for *all* individuals. A subtle but critical point is that actuarial fairness is impossible if there is a single DRC that applies to everyone and the pension system is redistributive. The following proposition formalizes this idea.

Proposition 1 *For the special case where the pension system is Bismarckian (rich and poor have the same replacement rate), then actuarial fairness for the rich would imply fairness for* *the poor and vice versa. For the more general case where the pension system is redistributive (the rich have a lower replacement rate than the poor), the DRC must be positively related to earnings to achieve actuarial fairness.*

Proof. Use Leibniz's rule to differentiate (12),

$$\frac{dV^i(T)}{dT} = \frac{db^i(T)}{dT}\int_T^{\bar{T}} S(t)dt - S(T)\left(b^i(T) + \theta w^i\right). \tag{13}$$

The first term on the right of (13) is the present value of the extra benefits that are due to the DRC. The second term is the cost of delayed retirement, which is the sum of the present value of lost benefits and the present value of the extra taxes paid. The DRC would be actuarially fair (NPV neutral) if these two terms are equal, i.e.,

$$\frac{dV^i(T)}{dT} = 0, \text{ for all } T \in [T_{\min}, T_{\max}] \text{ and for } i \in \{-,+\}. \tag{14}$$

Combining (13) and (14) gives a differential equation with variable coefficient and variable term

$$\frac{db^i(T)}{dT} = \lambda(T)b^i(T) + \lambda(T)\theta w^i, \text{ for } T \in [T_{\min}, T_{\max}] \text{ and } i \in \{-,+\}, \tag{15}$$

where

$$\lambda(T) \equiv S(T) \times \left[\int_T^{\bar{T}} S(t)dt\right]^{-1}. \tag{16}$$

Equation (15) defines the actuarially fair adjustments that need to be made to the benefits at each retirement age. The general solution to this differential equation, for a constant k, is

$$b^i(T) = \left(k + \int^T \lambda(Z)\theta w^i \exp\left[-\int^Z \lambda(U)dU\right]dZ\right)\exp\left[\int^T \lambda(Z)dZ\right], \tag{17}$$

where Z and U are dummies of integration. Evaluate (17) at $T = T_{\min}$,

$$k = b^i(T_{\min})\exp\left[-\int^{T_{\min}} \lambda(Z)dZ\right] - \int^{T_{\min}} \lambda(Z)\theta w^i \exp\left[-\int^Z \lambda(U)dU\right]dZ, \tag{18}$$

and insert (18) into (17) to get a particular solution

$$b^i(T) = b^i(T_{\min})\exp\left[\int_{T_{\min}}^T \lambda(Z)dZ\right] + \int_{T_{\min}}^T \lambda(Z)\theta w^i \exp\left[\int_Z^T \lambda(U)dU\right]dZ. \tag{19}$$

Equation (19) gives the timepath of the actuarially fair benefit annuity, relative to the value of the benefits at the earliest collection date $b^i(T_{\min})$. Define the actuarially fair DRC, Δ, as the cumulative percentage increase in the annuity value of fair benefits above the early retirement benefit annuity

$$\begin{aligned}\Delta^i(T) &\equiv \frac{b^i(T)}{b^i(T_{\min})} - 1 &(20)\\ &= \exp\left[\int_{T_{\min}}^T \lambda(Z)dZ\right] + \left(R^i\right)^{-1}\int_{T_{\min}}^T \lambda(Z)\theta\exp\left[\int_Z^T \lambda(U)dU\right]dZ - 1,\end{aligned}$$

where R^i is the replacement rate at the earliest age of eligibility

$$R^i \equiv \frac{b^i(T_{\min})}{w^i}. \tag{21}$$

If the pension system is Bismarckian, then $R^- = R^+$, and hence $\Delta^-(T) = \Delta^+(T)$ for all T. Thus, for the special case of a Bismarckian system, one DRC function would be actuarially fair for both the rich and poor. However, for the more general case where the system is redistributive, $R^- > R^+$, then $\Delta^-(T) < \Delta^+(T)$ because $\lambda(T) > 0$ for all $T \in [T_{\min}, T_{\max}]$. Actuarial fairness would require a different DRC function for the rich and the poor; it would be impossible to obtain fairness with a single DRC function. ■

Note that the issue is *not* that the rich need a bigger DRC because they pay more taxes in an absolute sense; instead, the issue is that the rich need a bigger DRC because they pay larger taxes relative to the benefits they receive. Think of the extreme example of a Beveridgean pension system to make the intuition clear. In this case, the rich pay more taxes but receive the same benefits as the poor. If both individuals delay retirement for one year, the rich will pay higher taxes during that extra year and therefore would need a higher percentage increase in benefits (a higher DRC) to compensate for the higher taxes paid.

Proposition 2 *The ratio of the fair DRC of the rich to the fair DRC of the poor is simple enough to calculate by hand because the ratio depends only on the replacement rates of the two groups.*

Proof. To show this, note that we can rewrite (20) by using the chain rule to express the second integrand differently

$$\Delta^i(T) = \exp\left[\int_{T_{\min}}^{T} \lambda(Z)dZ\right] + \left(R^i\right)^{-1}\theta \int_{T_{\min}}^{T} \left\{-\frac{d}{dZ}\exp\left[\int_{Z}^{T} \lambda(U)dU\right]\right\} dZ - 1, \tag{22}$$

which simplifies to a multiplicative expression

$$\Delta^i(T) = \left[1 + \left(R^i\right)^{-1}\theta\right]\left\{\exp\left[\int_{T_{\min}}^{T} \lambda(Z)dZ\right] - 1\right\}. \tag{23}$$

The actuarially fair DRC for the rich exceeds the DRC for the poor (in terms of percentage) by

$$\frac{\Delta^+(T)}{\Delta^-(T)} - 1 = \frac{1 + \left(R^+\right)^{-1}\theta}{1 + \left(R^-\right)^{-1}\theta} - 1, \text{ for any } T \in (T_{\min}, T_{\max}]. \tag{24}$$

Thus, for a given tax rate, the ratio of the DRCs is only related to the replacement rates of the two groups. With just these pieces of information only, (24) shows how much bigger the DRC of the rich would need to be to satisfy actuarial fairness. Equation (24) is so simple that it can be calculated by hand. No numerical approximation of any kind is required. ■

Proposition 3 *The difference between the fair DRC of the rich and the fair DRC of the poor is maximized in a Beveridgean pension system.*

Proof. The proof is simple and logical. Think of a spectrum of pension systems ranging from Bismarckian to Beveridgean. Holding the generosity of the system constant (holding contributions constant at θ), $(R^{+})^{-1}$ is monotone increasing and $(R^{-})^{-1}$ is monotone decreasing as we move from Bismarckian to Beveridgean, implying that (24) is maximized when the system is Beveridgean. ■

A final theoretical result deals with the importance of longevity.

Proposition 4 *An increase in life expectancy reduces the fair DRC for both the rich and the poor in partial equilibrium.*

Proof. Note that additional years of life expectancy, conditional on surviving to a given date of retirement T

$$\int_{T}^{\bar{T}} \frac{t}{S(T)} \left[\frac{-dS(t)}{dt} \right] dt - T \tag{25}$$

can be integrated by parts (Sheshinski (2008b))

$$\int_{T}^{\bar{T}} \frac{t}{S(T)} \left[\frac{-dS(t)}{dt} \right] dt - T = \frac{1}{S(T)} \int_{T}^{\bar{T}} S(t)dt = \lambda(T)^{-1}. \tag{26}$$

Note that delaying retirement by one year costs the individual θw^i in extra taxes paid and $b^i(T)$ in lost pension benefits. For the system to be actuarially fair, the pension annuity must be larger each period by $\theta w^i + b^i(T)$ divided by the conditional life expectancy $\lambda(T)^{-1}$, which is the same thing as saying that the annuity value of pension benefits must increase by $[\theta w^i + b^i(T)]\,\lambda(T)$ (see (15)). Thus, a greater life expectancy leads to a longer period over which the costs can be spread and hence the fair DRC would be smaller. More precisely, it is clear from (23), which is our definition of the fair DRC, that higher conditional life expectancy leads to a smaller DRC. ■

A caveat needs to follow the last proposition. The effect of increased longevity is ambiguous in general equilibrium because replacement rates (for both the rich and the poor) would fall in typical macroeconomic simulations. Therefore, in addition to the direct effect of increased longevity on the fair DRC (which is the focus of the previous proposition), in equilibrium there would also be an indirect, counter effect (though the first term in (23)). Understanding the precise effect of increased longevity is therefore a quantitative question for a full-blown, calibrated general equilibrium model.

4 A Numerical Example with US Bend Points

To show that the analytical results above are quantitatively meaningful, we begin by calibrating the parameters of the model to match features of the US social security system. We assume that model age $t = 0$ corresponds to age 25 when the individual first enters the workforce. The minimum eligibility age $T_{\min}$ is set to model age 37 (which corresponds to actual age 62) and $T_{\max}$ is set to 45 because no more rewards for delayed retirement are possible after age 70. We set the social security tax rate θ to the OASI part of the social security tax in the US (10.6%).

We want to know what the fair DRC would look like conditional on having a benefit-earning rule like we observe in the US. That is, in order to compute $b^i(T)$, we need to know $b^i(T_{\min})$, and we get these values by matching to the US system in order to make the scale of the calculations comparable to the US. Benefits in the US are a piecewise linear function of average wage income over the working period. The kinks in the function are called "bend points." The benefit function gets flatter at each bend point, which reflects redistribution of social security wealth from the elderly rich to the elderly poor. Social security replaces 90% of wage income up to the first bend point, 32% of income that is between the first and second bend points, 15% of income that is between the second and third bend points, and 0% of income that is beyond the third bend point. The actual values of the bend points change each year, but Ortiz (2009) and others assume the bend points are multiples of average earnings for the economy: 0.2, 1.24, and 2.47.

Simply as an example, we consider two individuals. One who makes twice the average wage income of the economy and the other who makes half the average (e.g., imagine that average annual wage income is \$40,000, while the rich person earns \$80,000 and the poor earns \$20,000). Normalizing the average to one, we have $w^+ = 2$ and $w^- = 0.5$. Using the US bend points and assuming retirement at the "normal" age, denoted T_{norm}, the benefits of the rich and poor would be

$$b^+(T_{\text{norm}}) = 0.2 \times 90\% + (1.24 - 0.2) \times 32\% + (2.0 - 1.24) \times 15\% = 0.6268, \tag{27}$$

$$b^-(T_{\text{norm}}) = 0.2 \times 90\% + (0.5 - 0.2) \times 32\% = 0.2760. \tag{28}$$

The normal retirement age for those retiring today in the US is 66. If early retirement at age 62 is chosen, benefits are reduced by 75% for the rich and poor alike. Thus,

$$b^+(T_{\min}) = 75\% \times b^+(T_{\text{norm}}) = 0.4701, \tag{29}$$

$$b^-(T_{\min}) = 75\% \times b^-(T_{\text{norm}}) = 0.2070. \tag{30}$$

These initial values reflect the degree of redistribution inherent in the US social security system and they also reflect the overall generosity of the US system. We could use any initial values we want, but we use (29) and (30) to make the calculations of the actuarially fair DRC comparable to the US. Note that the poor earn 25% of the wages of the rich, but the poor collect benefits which are 44% of the benefits of the rich. In a Bismarckian system, the poor benefits would be 25% of those of the rich. In a Beveridgean system, the poor benefits would be 100% of those of the rich. Because 44% is closer to 25% than to 100%, the US system is closer to Bismarckian than to Beveridgean. Using (29) and (30) to obtain the corresponding replacement rates, (24) tells us how much bigger the DRC needs to be for the rich to ensure actuarial fairness. We calculate $\Delta^+(T)/\Delta^-(T) - 1 = 16\%$.

Although the effect is already substantial, this estimate (i.e., 16%) will be even bigger if we increase the wage dispersion. Let us consider a case for which the dispersion in replacement rates is maximized. This happens if $w^+ = 2.47$ and $w^- = 0.2$, so that the incomes of the individuals are on the first and last bend points. In this case, $b^+(T_{\text{norm}}) = 0.6973$, $b^+(T_{\min}) = 0.5230$, and $R^+ = 0.2117$. Likewise, $b^-(T_{\text{norm}}) = 0.18$, $b^-(T_{\min}) = 0.135$, and $R^- = 0.675$. This gives $\Delta^+(T)/\Delta^-(T) - 1 = 30\%$. Thus, in the US economy the highest earners would need a DRC that is 30% larger than the DRC of the lowest earners to ensure actuarial fairness.

5 A Numerical Example with a Beveridgean Pension System

The degree of redistribution in pension systems varies across the OECD. The US program is closer to Bismarckian than to Beveridgean, but several other countries have systems that are more redistributive. The difference between the fair DRC for the rich and the fair DRC for the poor is most exaggerated in a Beveridgean pension system (recall Proposition 3).

Let us set $b^+(T_{\min}) = b^-(T_{\min}) = 0.3386$, which is the average of the initial values in (29) and (30). Let us also keep the initial assumption that $w^+ = 2$ and $w^- = 0.5$. These assumptions keep the scale of the model consistent with the results from the previous section. Now we get $\Delta^+(T)/\Delta^-(T) - 1 = 41\%$, meaning the DRC would need to be 41% higher for the rich than the poor in order to achieve actuarial fairness. This is a large difference, and it can be made even larger if we increase the wage dispersion among the two individuals under consideration. For example, using the wage dispersion in the last paragraph of the previous section, we get 69% instead of 41%.

6 Summary

We have shown theoretically that the Delayed Retirement Credit (DRC) would need to be higher for the rich than for the poor in order to achieve actuarially fairness. Our numerical exercises confirm that the magnitude of the difference is significant, especially as the pension systems tends toward a Beveridgean pension. Public pension designers may want to consider linking the DRC positively to earnings to avoid distortions to retirement which will necessarily arise if the DRC is the same for all individuals. We have reached these conclusions by analyzing a differential equation with variable coefficient and variable term.

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