Optimal Social Security Reform under Population Aging in the U.S.*

Shantanu Bagchi†
Utah State University
shantanu.bagchi@aggiemail.usu.edu
http://shan.bagchi.googlepages.com

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Abstract

Actuaries of the Social Security Administration (SSA) estimate that due to population aging in the U.S., the current Old-Age and Survivors Insurance (OASI) tax rate may have to be increased from 10.6% to 15.7% in the year 2075 to prevent any decline in the projected benefits. In this paper I ask what should be the optimal or welfare-maximizing OASI tax rate in the U.S. under such demographic developments. I examine this question using a heterogeneous-agent general equilibrium model of life-cycle consumption and labor supply, where social security provides partial insurance against unfavorable efficiency realizations that occur before the agents enter the model. I calibrate the model such that the current OASI tax rate in the U.S. maximizes social welfare under the current demographics, incorporate empirically reasonable population projections, and then search for the tax rates that are optimal under such projections. I find that the tax rate that maximizes welfare under such projections is about 2 to 5 percentage points higher than the current rate. I also find that a large part of the tax burden of population aging is picked up by the households with relatively favorable efficiency realizations. Finally, the model also predicts that population aging and the optimal tax response may imply a decline in the projected retirement benefits.

JEL Classification: E21, H55, J26

Keywords: Social security; Heterogeneous agents; Insurance; Population aging; Welfare maximization; Life-cycle consumption; Labor supply; General equilibrium

1 Introduction

All across the OECD, lower birth rates and higher life expectancies have threatened the viability of unfunded social security programs. In the U.S., actuaries of the Social

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Security Administration (SSA) estimate that in the year 2075, a 33% decline in the projected retirement benefits is required to keep the current program solvent with the existing contribution rate. They also estimate that to prevent any decline in the projected benefits for that year, the current Old-Age and Survivors Insurance (OASI) tax rate may have to be increased to 15.7%.

In this paper I ask what should be the optimal or welfare-maximizing OASI tax rate in the U.S. under the population developments projected in the future. There are at least two reasons why this question is not trivial. First, even though it is intuitive that a higher tax rate may be required to balance the social security budget under population aging, it is not clear what impact such a strategy would have on social welfare: a higher tax rate can potentially distort individual behavior and equilibrium factor prices in a way that reduces utility. Second, whether or not social security improves welfare depends on which missing market it substitutes. If individuals face uninsurable productivity realizations that occur prior to their entering the labor force, and if social security already insures them against this through a pro-poor retirement benefit rule, then it is not clear whether population aging would create the need for a larger program.

To examine this issue, I begin by constructing a heterogeneous-agent general equilibrium model of life-cycle consumption and labor supply, where the source of heterogeneity is a productivity or efficiency realization that occurs before the agents enter the model. In the model, an unfunded social security program provides partial insurance against an unfavorable efficiency realization by paying retirement benefits through a pro-poor rule. Agents in the model also face mortality risk, against which they cannot insure because of the absence of private annuity markets. I first calibrate the benefit rule to match the degree of redistribution in the U.S. social security program, and then I calibrate the distribution of efficiency such that the current OASI tax rate in the U.S. maximizes social welfare under the current demographics. Then, I introduce empirically reasonable future population projections into the calibrated model, and finally I search for the tax rates that maximize social welfare under those projections.

The baseline equilibrium of the model performs reasonably well in matching some key features of the current U.S. economy, such as the aggregate capital-output ratio, the average fraction of time spent on market work, and the gross benefit replacement rates in the population. Also, in the baseline equilibrium, social security is welfare-improving only for the households with relatively poor efficiency realizations. The relatively efficient households experience welfare losses from social security: their internal rates of return from the program are lower than the market rate of return on capital stock.

The main findings of this paper are as follows. First, I find that the optimal or welfare-maximizing OASI tax rate under the future population projections in the U.S. is about 2 to 5 percentage points higher than the current tax rate. Second, I also find that households with different efficiency realizations respond asymmetrically to the the demographic developments, and that a large part of the tax burden of population aging is picked up by the households with relatively favorable efficiency realizations. Finally, the model also predicts that population aging and the optimal tax response may imply a decline in the projected retirement benefits.

Among others, three important quantitative studies on social security reform under population aging in the U.S. are De Nardi et al. (1999), Conesa and Garriga (2008) and Conesa and Garriga (2009). The current paper complements these studies in two ways. First, I employ a heterogeneous-agent model to study optimal social security reform, where social security provides partial insurance against unfavorable efficiency realizations through a pro-poor retirement benefit rule. This allows me to replicate the degree of redistribution in the U.S. social security program, as the benefit replacement rate of a household in the U.S. is a concave function of work-life income.[1] Second, I only consider equilibria of the model in which the OASI tax rate maximizes social welfare. This allows me to run controlled demographic experiments in which the welfare-maximizing changes in the tax rate predicted by the model can be fully attributed to population aging. To accomplish this, I first calibrate the efficiency distribution in the baseline model such that the current OASI tax rate in the U.S. is exactly optimal, and then I introduce empirically reasonable future population projections. Note that controlling for the optimal program size under the current demographics is crucial, as failing to do so can potentially confound the optimal tax response to population aging.

Beginning with Feldstein (1985), a number of studies have attempted to justify the size of the current social security program in the U.S. on the grounds of its different welfare-improving roles.[2] However, studies in this area have typically focused on computing the optimal or welfare-maximizing social security tax rate under the current U.S. demographics.[3] The current paper also complements these studies, as I compute the optimal OASI tax rate under the future demographic projections in the U.S., conditional on the assumption that the current tax rate is exactly optimal.

The rest of the paper is organized as follows: Section 2 introduces the model, Section 3 describes the baseline calibration, Section 4 examines the impact of empirically reasonable future population projections on the calibrated model, and Section 5 concludes.

2 The model

Consider a continuous time overlapping generations economy with life-cycle permanent income households, where at each instant a new cohort is born and the oldest cohort dies. Cohorts are identical in all respects but their date of birth, but within each cohort there is heterogeneity with respect to household efficiency. The maximum lifespan of a household is $\bar{T}$ years, and the life cycle consists of two phases: work and

[1] Huggett and Ventura (1999) investigate social security reform in the U.S. with a two-tier structure using a heterogeneous-agent model with the concave Primary Insurance Amount (PIA) benefits formula used by the SSA.

[2] See, for example studies such as İmrohoroğlu et al. (1995), İmrohoroğlu et al. (2003), Caliendo and Gahramanov (2009) and Findley and Caliendo (2009), among others.

[3] An exception is found in Findley and Caliendo (2009), who find in a robustness analysis that the average tax rate in the presence of short planning horizons under future demographics increases slightly from 11 percent in their baseline to about 12 percent.

retirement. During the final $\bar{T} - T_{\rm r}$ years of life, the household receives social security benefits that are positively related to their work life income. Households face mortality risk against which they cannot insure because of closed private annuity markets, and they derive utility from consumption (c) as well as the fraction of total time endowment enjoyed in leisure (l). They also accumulate a risk-free asset: physical capital. The government runs an unfunded social security program that is financed through taxes on labor income, and the assets of the deceased households at each instant are uniformly distributed over the surviving population in the form of accidental bequests. Perfectly competitive firms produce output using a constant returns to scale Cobb-Douglas production function with constant labor-augmenting technological progress at rate g per annum, and there is no aggregate uncertainty. Finally, population grows at rate n per annum.

2.1 Preferences

The period utility function is

$$u(c, l) = \begin{cases} \frac{\left(c^{\eta} l^{1-\eta}\right)^{1-\sigma}}{1-\sigma} & \text{if} \quad \sigma \neq 1 \\ \ln\left(c^{\eta} l^{1-\eta}\right) & \text{if} \quad \sigma = 1 \end{cases} \tag{1}$$

where η is the share of consumption in period utility and σ is the Inverse Elasticity of Intertemporal Substitution (IEIS). Expected lifetime utility from the perspective of a household at age $s = 0$ is

$$U = \int_0^{\bar{T}} \exp\left\{-\rho s\right\} Q(s) \frac{\left\{c(s)^{\eta} l(s)^{1-\eta}\right\}^{1-\sigma}}{1-\sigma} \mathrm{d}s \tag{2}$$

where ρ is the discount rate and $Q(s)$ is the unconditional probability of surviving to age s. Also, since I define leisure as a fraction of the total time endowment, we have $0 \leq l(s) \leq 1$.

2.2 Income

Each household in a given cohort is endowed with an efficiency endowment $\varphi e(s)$, where φ is a realization from a stationary distribution with density $f(\varphi)$ and support $\left[\underline{\varphi}, \overline{\varphi}\right]$ that occurs prior to birth, and $e(s)$ is an age-dependent component that increases early in life, peaks at about middle age, and then declines until death. Households' saving $a(s)$ earns a real rate of return r, and during the work life labor income is taxed at rate θ, which is the OASI tax rate. The tax receipts are used to pay social security benefits to households past the eligibility age of $T_{\rm r}$. The surviving households also receive an accidental bequest $B(t)$ from the deceased households every period.

2.3 Social security

The government runs an unfunded social security program that partially insures households against unfavorable efficiency realizations through a pro-poor benefit rule. The benefit annuity at date t of a household with efficiency φ is $b(t;\varphi)$, which is calculated as follows:

$$b(t;\varphi) = \zeta(\varphi) b(t;\overline{\varphi}) \tag{3}$$

$$\zeta(\overline{\varphi}) = 1 \tag{4}$$

$$\zeta(\varphi) = \left[\frac{\zeta(\underline{\varphi})\overline{\varphi} - \underline{\varphi}}{\overline{\varphi} - \underline{\varphi}}\right] + \left[\frac{1 - \zeta(\underline{\varphi})}{\overline{\varphi} - \underline{\varphi}}\right]\varphi \tag{5}$$

where $b(t;\overline{\varphi})$ is the retirement benefit paid at date t to the households with the highest efficiency realization, $\zeta(\varphi)$ is a linear function of efficiency φ, and $\zeta(\underline{\varphi})$ is a parameter that controls the extent of redistribution in the social security program. As Caliendo and Gahramanov (2009) have shown, for the social security program to be both pro-poor and positively linked to past income with the benefit rule outlined in (3), we must have

$$\underline{\varphi} \leq \zeta(\underline{\varphi}) \leq 1 \tag{6}$$

Also, since I only consider equilibria of the model where the OASI tax rate is optimal or maximizes social welfare, θ must satisfy

$$\theta = \text{arg max}\ \left\{\int_{\underline{\varphi}}^{\overline{\varphi}} U(\varphi)\ f(\varphi)\ \mathrm{d}\varphi\right\} \tag{7}$$

where $U(\varphi)$ is the ex-ante expected lifetime utility of households with efficiency φ.

2.4 Household optimization problem

A household born at date t with the efficiency realization φ faces the following optimization problem

$$\max_{c(s),l(s)} \int_0^{\bar{T}} \exp\{-\rho s\} Q(s) \frac{\left\{c(s)^{\eta} l(s)^{1-\eta}\right\}^{1-\sigma}}{1-\sigma}\, \mathrm{d}s \tag{8}$$

subject to

$$c(s) + \frac{\mathrm{d}a(s)}{\mathrm{d}s} = ra(s) + y(s;\varphi) + B(t)\exp\{gs\} \tag{9}$$

$$y(s;\varphi) = (1-\theta)\{1 - l(s)\} w(t)\exp\{gs\}\varphi e(s) + \Theta(s - T_{\mathrm{r}}) b(t;\varphi)\exp\{gs\} \tag{10}$$

$$0 \leq l(s) \leq 1 \tag{11}$$

$$a(0) = a(\bar{T}) = 0 \tag{12}$$

where

$$\Theta = \begin{cases} 0 & x \leq 0 \\ 1 & x > 0 \end{cases}$$

is a step function. I solve this problem using Pontryagin's Maximum Principle for fixed-endpoint optimal control problems.

2.5 Technology and factor prices

Output is produced using a Cobb-Douglas production function with inputs capital, labor and a stock of technology $A(t)$

$$Y(t) \;=\; K(t)^{\alpha}\,(A(t)L(t))^{1-\alpha} \tag{13}$$

where $A(t) = A(0)\exp\{gt\}$, α is the share of capital in total income and $A(0)$ is the initial stock of technology. Factor markets are perfectly competitive and equilibrate instantaneously, which implies

$$r = MP_K - \delta = \alpha\left[\frac{K(t)}{A(t)L(t)}\right]^{\alpha-1} - \delta \tag{14}$$

$$w(t) = MP_L = A(t)(1-\alpha)\left[\frac{K(t)}{A(t)L(t)}\right]^{\alpha} \tag{15}$$

where δ is the depreciation rate of physical capital and $w(t)$ is the wage rate at time t.

2.6 Aggregation

Aggregate capital stock and labor supply in the current model are given by

$$K(t) = \int_{\underline{\varphi}}^{\overline{\varphi}} \int_0^{\bar{T}} N(t-s)Q(s)\,a(t;t-s,\varphi)\,f(\varphi)\,\mathrm{d}s\,\mathrm{d}\varphi \tag{16}$$

$$L(t) = \int_{\underline{\varphi}}^{\overline{\varphi}} \int_0^{\bar{T}} N(t-s)Q(s)\;\{1 - l(t;t-s,\varphi)\}\,\varphi e(s)\,f(\varphi)\,\mathrm{d}s\,\mathrm{d}\varphi \tag{17}$$

where $N(t-s)$ is the size of the cohort born at date $(t-s)$. Also, given that the social security program is unfunded, total taxes collected at any date must be equal to the total benefits paid out, i.e.

$$\begin{aligned}
&\int_{\underline{\varphi}}^{\overline{\varphi}} \int_0^{\bar{T}} N(t-s)Q(s)\theta\,\{1 - l(t;t-s,\varphi)\}\,w(t)\varphi e(s)\,f(\varphi)\,\mathrm{d}s\,\mathrm{d}\varphi \\
&\qquad = \int_{\underline{\varphi}}^{\overline{\varphi}} \int_{T_{\mathrm{r}}}^{\bar{T}} N(t-s)Q(s)b(t;\varphi)\,f(\varphi)\,\mathrm{d}s\,\mathrm{d}\varphi
\end{aligned} \tag{18}$$

Using (3), we can rearrange and express (18) as

$$b(t;\overline{\varphi}) = \theta w(t)\left[\frac{\int_{\underline{\varphi}}^{\overline{\varphi}} \int_0^{\bar{T}} N(t-s)Q(s)\,\{1 - l(t;t-s,\varphi)\}\,\varphi e(s)\,f(\varphi)\,\mathrm{d}s\,\mathrm{d}\varphi}{\int_{\underline{\varphi}}^{\overline{\varphi}} \int_{T_{\mathrm{r}}}^{\bar{T}} N(t-s)Q(s)\,\zeta(\varphi)\,f(\varphi)\,\mathrm{d}s\,\mathrm{d}\varphi}\right] \tag{19}$$

which expresses the retirement benefits paid to the most efficient households in the population as a function of relevant macroeconomic and demographic variables. I define the box-bracketed term in equation (19) as the labor-to-retiree ratio and denote it by the symbol $R^e(t)$. Finally, total accidental bequests from the deceased households must satisfy

$$\int_{\underline{\varphi}}^{\overline{\varphi}} \int_0^{\bar{T}} N(t-s)Q(s)h(s)a(t;t-s,\varphi)\, f(\varphi)\,\mathrm{d}s\,\mathrm{d}\varphi$$
$$= \int_{\underline{\varphi}}^{\overline{\varphi}} \int_0^{\bar{T}} N(t-s)Q(s)B(t)\, f(\varphi)\,\mathrm{d}s\,\mathrm{d}\varphi \tag{20}$$

where

$$h(s) = -\frac{\mathrm{d}}{\mathrm{d}s}\ln Q(s) \tag{21}$$

is the hazard rate of dying between age s and $s + \mathrm{d}s$.

2.7 Equilibrium

A Stationary Competitive Equilibrium (SCE) with an optimal OASI tax rate in the current model can be characterized by a collection of

1. cross-sectional consumption programs $\{c(t;t-s,\varphi)\}_{s=0}^{\bar{T}}$, saving programs $\{a(t;t-s,\varphi)\}_{s=0}^{\bar{T}}$ and labor supply programs $\{1-l(t;t-s,\varphi)\}_{s=0}^{\bar{T}}$ for each φ,
2. aggregate capital stock $K(t)$, labor supply $A(t)L(t)$ and labor-to-retiree ratio $R^e(t)$,
3. real rate of return r and wage rate $w(t)$,
4. a tax rate θ, and
5. an accidental bequest $B(t)$

that

1. solves the households' optimization problems,
2. equilibrates the factor markets and balances the social security budget,
3. satisfies the social welfare maximization condition (7), and
4. satisfies the bequest balance condition (20)

I assume that the model economy is initially at a SCE with an optimal OASI tax rate. It is useful to note that for this economy, along the steady state growth path aggregate output grows at rate $(n+g)$, the real rate of return is time-invariant and wages grow at rate g.

2.8 Computational algorithm

To compute the SCE for a given set of model parameters and an OASI tax rate, I use the following algorithm:

- Step 1: Guess some values for the factor prices, the labor-to-retiree ratio and the accidental bequest.
- Step 2: Solve the households' optimization problems under the values guessed in step 1.
- Step 3: Aggregate the household-level optimal choices to obtain the implied total capital stock and labor supply.
- Step 4: Compute the factor prices, the labor-to-retiree ratio and the accidental bequest implied by the values obtained in steps 2 and 3.
- Step 5: Repeat steps 1-4 until the guessed values in step 1 converge to the implied values in step 4.

Then, to find the optimal tax rate, I repeat steps 1-5 over a grid of tax rates and then choose the value that maximizes social welfare (i.e. satisfies condition (7)).

3 Baseline calibration

I parameterize the baseline equilibrium of the model using empirical evidence from various sources. A population growth rate of $n = 1\%$ is consistent with the U.S. demographic history, and I set the rate of technological progress to $g = 1.56\%$, which is the trend growth rate of per-capita income in the postwar U.S. economy (Bullard and Feigenbaum, 2007). I assume that households enter the model at actual age 25, which corresponds to the model age of zero. I obtain the survival probabilities from Feigenbaum's (2008) sextic fit to the mortality data in Arias (2004), which is given by

$$\begin{aligned}\ln Q(s) \quad = \quad & -0.01943039 + \left(-3.055 \times 10^{-4}\right) s + \left(5.998 \times 10^{-6}\right) s^2 \\ & + \left(-3.279 \times 10^{-6}\right) s^3 + \left(-3.055 \times 10^{-8}\right) s^4 + \left(3.188 \times 10^{-9}\right) s^5 \\ & + \left(-5.199 \times 10^{-11}\right) s^6 \end{aligned} \tag{22}$$

where s is model age. The 2001 U.S. Life Tables in Arias (2004) are reported up to actual age 100, so I set the maximum model age to $\bar{T} = 75$. Also, I set the model benefit eligibility age to $T_{\mathrm{r}} = 41$, which corresponds to the current actual full retirement eligibility age of 66 in the U.S. As the household's age-dependent efficiency endowment $e(s)$ is difficult to observe, I use average cross-sectional hourly income data from the 2001 CPS as a proxy for efficiency. To use this data, I first use piecewise linear interpolation to obtain average hourly earnings for all ages between 25-65, and normalize the data such that earnings at actual age 25 is unity. Then, I fit a quartic polynomial to the interpolated data, which gives

$$\begin{aligned}\ln e(s) &= -3.273\times10^{-5} + \left(1.423\times10^{-4}\right)s + \left(-3.8696\times10^{-5}\right)s^2 \\ &\quad + \left(-1.313\times10^{-5}\right)s^3 + \left(6.307\times10^{-8}\right)s^4 \end{aligned} \tag{23}$$

where s is model age and $s \leq 40$. Beyond actual age 65 (i.e. for $s > 40$), for which data is limited, I use the following quadratic function

$$\ln e(s) = -f_0 - f_1 s - 0.01s^2 \tag{24}$$

and parameterize f_0 and f_1 such that $e(s)$ is continuous and once differentiable at age $s = 40$.[4] The resulting efficiency profile is plotted in Figure 1.

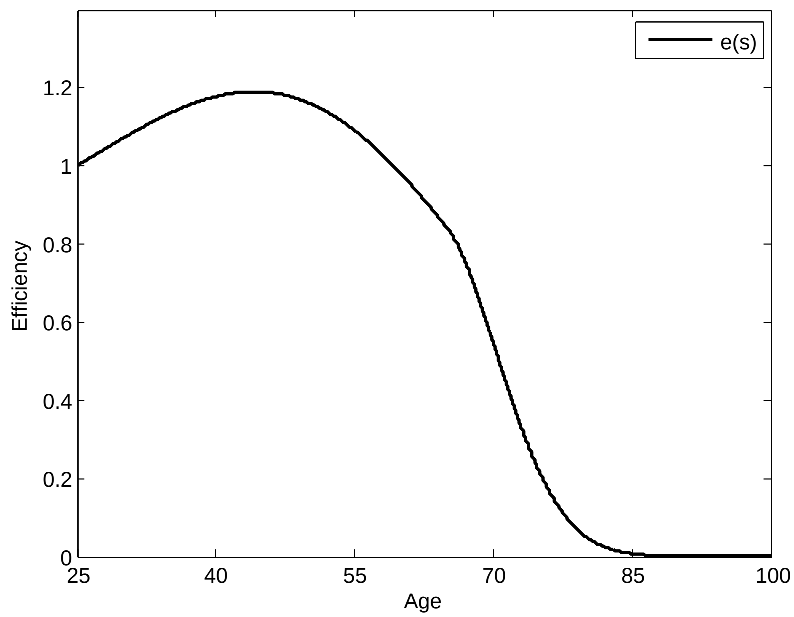

Figure 1: Efficiency profile estimated from the 2001 CPS.

The historically observed value of capital's share in total income in U.S. ranges between 30-40%, so I set $\alpha = 0.35$. Also, since I focus only on SCE, I set $t = 0$ in all the computations and normalize the initial stock of technology and the population to $A(0) = N(0) = 1$. I also assume that the random component of household efficiency (φ) is distributed uniformly within a cohort, and then transform the continuous distribution into a 5-point discrete distribution for computational convenience (i.e. $f(\varphi) = 0.2$).[5] To express efficiency relative to the most efficient households in the cohort (those with

[4]The values that satisfy these conditions are $f_0 = 14.7416$ and $f_1 = -0.7643$.

[5]Note that the 5-point specification also facilitates reporting model data by income quintiles.

$\varphi = \overline{\varphi}$), I specify the lower limit of the support as a fraction of the upper limit, i.e. $\underline{\varphi} = \omega\overline{\varphi}$ with $0 < \omega \leq 1$, and then normalize the upper limit to $\overline{\varphi} = 1$. Note that with this specification, the degree of efficiency heterogeneity within a cohort can be conveniently controlled by simply varying the parameter ω. Finally, I set the depreciation rate to $\delta = 0.0744$.

Once all the observable parameters have been assigned empirically reasonable values, I calibrate the unobservable preference parameters σ (IEIS), ρ (discount rate) and η (share of consumption in period utility), the efficiency heterogeneity parameter ω and the benefit rule parameter $\zeta(\omega)$ (which controls the degree of redistribution in the social security program) such that the model jointly matches the following targets:

- a steady state capital-output ratio of 3.0,
- an average fraction of time of $34/168 = 0.205$ spent on market work between ages 25-55,
- a replacement rate of 90% for the poorest households in the population, and
- an optimal or welfare-maximizing OASI tax rate of 10.6%.[6]

The capital-output ratio target is consistent with the larger macroeconomic literature. The target for the fraction of time spent on market work is taken from the 2001 CPS, which reports that on an average, production and nonsupervisory employees in the U.S. spend 34 hours per week on market work. The replacement rate target is taken from the Primary Insurance Amount (PIA) benefit formula used by the SSA, which replaces 90 percent of the average indexed monthly earnings among the poorest income earners in the population. Finally, the current OASI tax rate target allows me to fully control for the optimal program size under the current U.S. demographics.

The parameter values under which the model reasonably matches the above targets are reported in Table 1. Note that discount rates close to zero or even negative are not uncommon in the macro-calibration literature (Huggett, 1996; Bullard and Feigenbaum, 2007; Feigenbaum, 2008) as well as the quantitative public finance literature (Huggett and Ventura, 1999; Conesa and Garriga, 2008, 2009), and a share of consumption in period utility around one-sixth is reasonably close to the values used in the general pension reform literature (Kotlikoff, 1997; Huggett and Ventura, 1999; Nishiyama and

[6] I compute the replacement rate at date t for a household with efficiency φ surviving to the eligibility age as follows. First, I compute the average indexed pre-tax earnings using the formula

$$AIE(t;\varphi) = \left\{ \int_0^{T^*(\varphi)} \{1 - l(t; t-s, \varphi)\} \, w(t)\varphi e(s) \, \mathrm{d}s \right\} / T^*(\varphi)$$

where $T^*(\varphi)$ is the retirement age of households with efficiency φ, or the age at which labor supply drops to zero. Note that similar to the SSA's calculations, I index past wages to date t in computing the AIE. Then, the I compute the replacement rate using the formula

$$RR(t;\varphi) = b(t;\varphi)/AIE(t;\varphi)$$

Table 1: Unobservable parameter values under the baseline calibration.

σ	ρ	η	ω	$\zeta(\omega)$
1.9	-0.0015	0.176	0.2	0.46

Smetters, 2005). Also, note that with leisure in period utility, the relevant IEIS for consumption is $\sigma^c = 1 + \eta(\sigma - 1) = 1.16$, which lies conveniently within the range frequently encountered in the literature.

The factor prices in the baseline equilibrium are $r = 0.0293$ and $w = 1.25$, the labor-to-retiree ratio is $R^e = 0.6386$, and the accidental bequest is $B = 0.0015$. I report the model-generated values for the targets under the baseline calibration in Table 2. The

Table 2: Model performance under the baseline calibration.

	Target	Model
Capital-output ratio	3.0	3.37
Avg. fraction of work time between ages 25-55	0.205	0.205
Replacement rate for the poorest households	0.9	0.896
Social security tax rate	0.106	0.107

baseline equilibrium cross-sectional age-consumption and age-labor hour profiles for the different efficiency groups are reported in Figures 2 and 3. It is useful to note that the average cross-sectional consumption profile (not plotted in Figure 2) exhibits a peak at about age 51, with a ratio of peak to initial consumption of 1.22. Also note that in the figures, 1 corresponds to the lowest efficiency level and 5 corresponds to the highest.

Since households in the current model are life-cycle permanent income consumers, a qualitative measure of their welfare gains from the social security program can be obtained by comparing their internal rates of return (IRR) from the program with the equilibrium rate of return from the capital stock. By definition, the IRR of a household with efficiency φ is the discount rate $\beta(\varphi)$ for which the net present value of the household's social security wealth equals zero. Mathematically, it is a solution to the following equation

$$\int_0^{\bar{T}} \exp\left\{(g - \beta(\varphi))s\right\} Q(s)\theta\left\{1 - l(s;\varphi)\right\} w(t)\varphi e(s)\,\mathrm{d}s = \int_{T_{\mathrm{r}}}^{\bar{T}} \exp\left\{(g - \beta(\varphi))s\right\} Q(s) b(t;\varphi)\,\mathrm{d}s \qquad (25)$$

where s is model age. Note that the first term in the above equation gives the present value of total benefits received and the second term gives the present value of the total tax payments. Numerically solving equation (25) for each efficiency group yields the following IRR distribution under the baseline calibration: $\{0.0479, 0.0323, 0.0256, 0.0218, 0.0192\}$. Comparing these IRRs with the equilibrium rate of return from capital stock shows that the bottom two quintiles of the model population experience a welfare

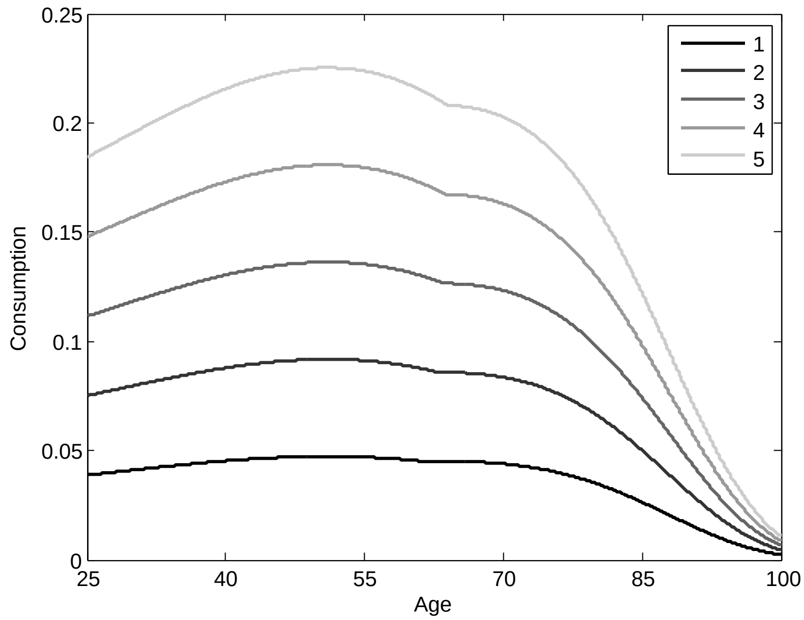

Figure 2: Baseline cross-sectional age-consumption profiles by efficiency level.

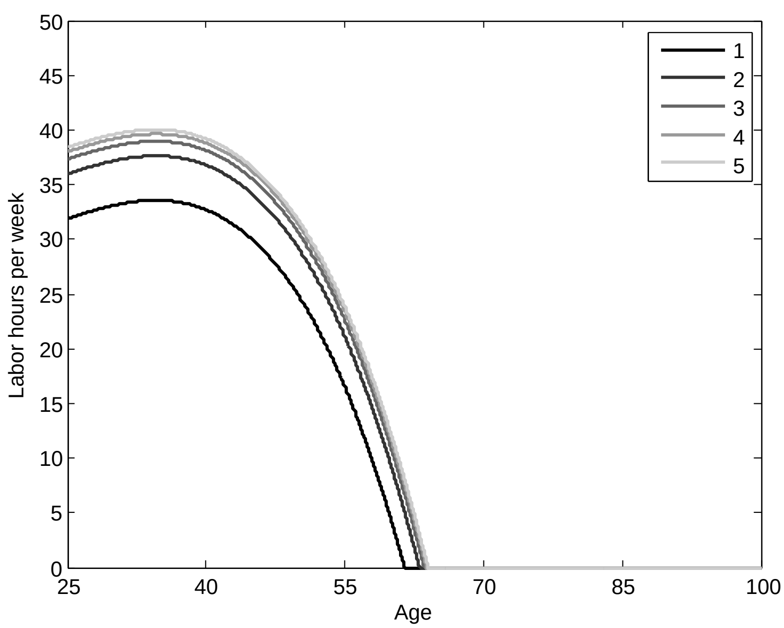

Figure 3: Baseline cross-sectional age-labor hour profiles by efficiency level.

gain from participating in the social security program, and the top three quintiles experience a welfare loss. With $\zeta(\omega) = 0.46$, the social security program partially insures the households against unfavorable efficiency realizations: the benefit annuity of the poorest households is close to half of that of the wealthiest, when the labor income of the former is only one-fifth of the latter (as $\omega = 0.2$). The gross benefit replacement rates in the model population under the baseline calibration are compared to the ones implied by the U.S. PIA benefits formula in Figure 4. The optimal retirement age

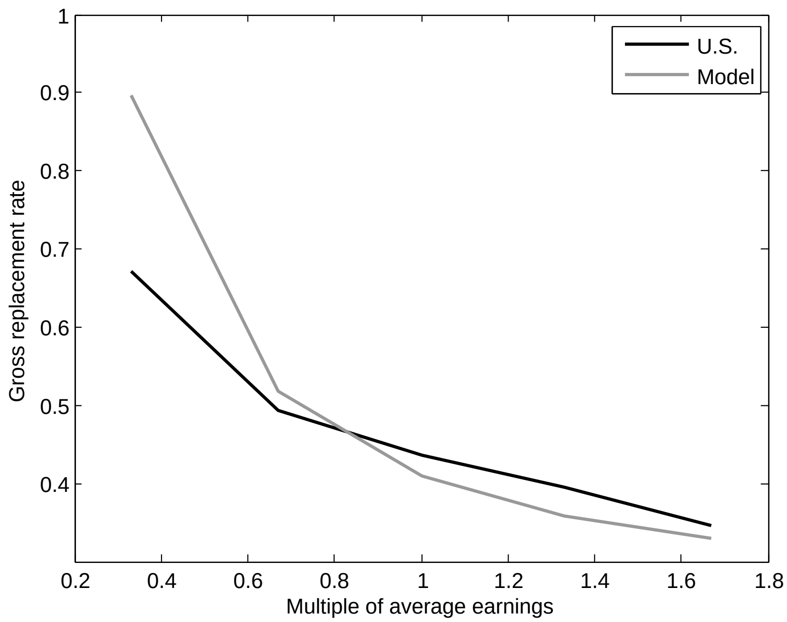

Figure 4: Gross replacement rates: Model Vs. U.S.

distribution in the baseline calibration is $\{61.48, 63.09, 63.61, 63.86, 64.02\}$, which shows that households with lower efficiency retire earlier than those with higher efficiency.

4 Population aging

Population aging in the current model is driven by an increasing life expectancy and a falling population growth rate. The 2009 OASDI Trustees Report published by the SSA contains the long-range social security area population and the average life expectancy at birth projections for 2075 under three plausible sets of assumptions: low-cost, i.e. favorable demographic conditions, intermediate, i.e. best estimates of likely future demographic conditions, and high-cost, i.e. financially disadvantageous demographic conditions. The projections are as follows:

1. Low-cost: life expectancy = 80.55, growth rate of the social security area population = 0.16%,

2. Intermediate: life expectancy = 83.95, growth rate of the social security area population = 0.09%, and

3. High-cost: life expectancy = 87.55, growth rate of the social security area population = 0.01%.

The projections for the growth rate of the social security area population can be taken directly to the model. To use the life expectancy projections, I augment the baseline survival probabilities with an age-specific increment of the form

$$\mathrm{d}\,Q(s) = \gamma s^{\mu} \tag{26}$$

where γ and μ are positive constants to be parameterized, and s represents household age. Therefore, the projected survival probabilities are given by

$$Q_{\mathrm{p}}(s) = Q(s) + \gamma s^{\mu} \tag{27}$$

Note that these age-specific increments are consistent with the fact that old-age survivorship in the U.S. has increased at a faster rate in the later half of the twentieth century, making the population survival curve more rectangular (Arias, 2004). Then, I choose values for γ and μ such that the model life expectancies under the augmented survival probabilities match the projections described above. I define the specific demographic experiments in Table 3.[7] The survivor functions corresponding to the three

Table 3: The demographic experiments.

Experiment	$n(\%)$	γ	μ	Life expectancy
Low-cost (1)	0.16	1×10^{-4}	1.5577	80.55
Intermediate (2)	0.09	6×10^{-5}	1.9077	83.95
High-cost (3)	0.01	4×10^{-5}	2.1298	87.55

experiments are compared to the baseline in Figure 5.

I incorporate these population projections into the baseline model, and then compute a new SCE with an optimal OASI tax rate under each experiment. I report the corresponding values of the social security tax rate and some other relevant variables in Table 4.

The results of the three experiments can be summarized as follows. First, the optimal or welfare-maximizing social security tax rate increases from the baseline level by 1.8 percentage points under the low-cost projection, 3.2 percentage points under the intermediate projection, and 4.8 percentage points under the high-cost projection. Second, capital accumulation increases and reduces the equilibrium rate of return, with 0.42, 0.71 and 0.99 percentage point declines under experiments 1, 2 and 3 respectively. Finally, the replacement rate for the lowest efficiency group declines to about 74, 71 and 68 percent under the three experiments, and the output level, labor supply and the capital-output ratio all increase.

[7] I hold the maximum lifespan unchanged at $\bar{T} = 75$ under all the three experiments.

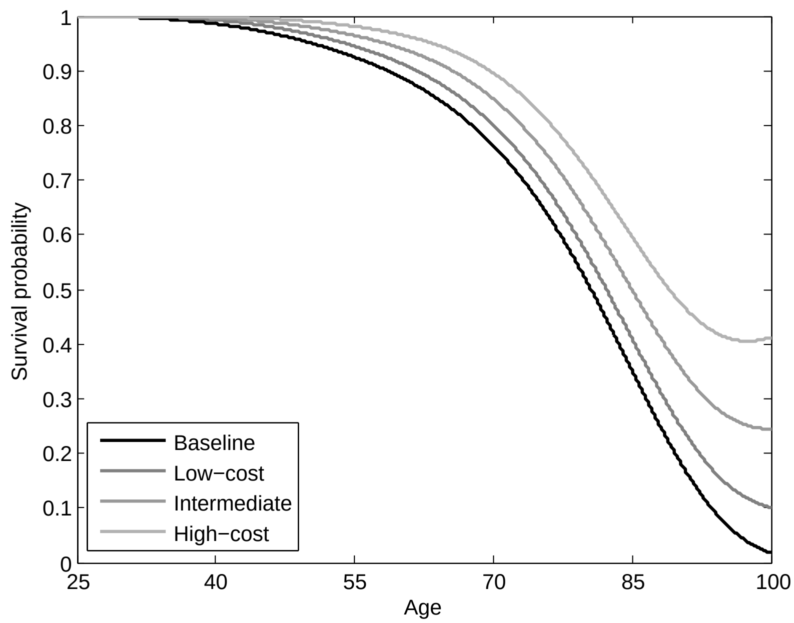

Figure 5: Baseline and the projected survival probabilities.

Table 4: The effect of population aging on the calibrated model.

Experiment	Baseline	1	2	3
OASI tax rate	0.107	0.125	0.139	0.155
Replacement rate for the poorest households	0.896	0.736	0.706	0.68
Rate of return	0.0293	0.0251	0.0222	0.0194
Wage rate	1.25	1.28	1.3	1.32
Output	7.55	9.2	9.86	10.56
Capital	25.47	32.36	35.75	39.41
Labor	3.92	4.67	4.93	5.2
Capital-output ratio	3.37	3.52	3.62	3.73

It is useful to compare the results in Table 4 to those in the other studies on social security reform under population aging in the U.S. De Nardi et al. (1999) consider eight alternative budget-balancing fiscal responses to future demographic shocks, which include keeping the benefits fixed and allowing the tax rates to adjust to the burden of demographic shocks, allowing the benefits to fall by increasing the retirement eligibility age, changing their tax treatment, or changing the benefit formula to allow for a larger dependence of benefits to past income. They find that allowing only the labor income tax rate to adjust to the shock requires it to increase by almost 30 percentage points to keep the benefits unchanged. However, when the benefits are allowed to fall from their baseline level, relatively smaller but still fairly large increases in the tax rate are required (of the order of 13 to 23 percentage points). On the other hand, Conesa and Garriga (2009) find that the average effective tax rate on labor income (which includes both the regular labor income tax as well as a payroll tax collected to finance social security) actually falls from its initial steady state level of 24.8 percent to around 22 percent. In comparison, the current model predicts that the optimal or welfare-maximizing response to population aging is likely to include tax increases ranging from roughly 2 to 5 percentage points. Also, it is important to note that these increases in the tax rate are always smaller than the actuarial projections of the SSA.

Why do the optimal or welfare-maximizing responses to population aging require relatively small increases in the tax rate? To understand this, I examine how population aging affects the extent to which households with different efficiency levels benefit from the social security program. In Table 5, I report the households' IRRs from social security under the three experiments, while holding the tax rate fixed at the baseline level. Note that the rates of return reported in Table 5 are the equilibrium values in post-population aging SCE with $\theta = 0.107$. The table shows that population aging

Table 5: The effect of the demographic shocks on the households' IRRs from social security.

Experiment	$\beta_{\varphi=0.2}$	$\beta_{\varphi=0.4}$	$\beta_{\varphi=0.6}$	$\beta_{\varphi=0.8}$	$\beta_{\varphi=1}$	Rate of return
Baseline	0.0479	0.0323	0.0256	0.0218	0.0192	0.0293
1	0.0406	0.0242	0.0172	0.0132	0.0106	0.0237
2	0.0399	0.0235	0.0165	0.0125	0.0099	0.0199
3	0.0391	0.0227	0.0157	0.0118	0.0092	0.0161

negatively impacts the IRRs from social security across all the efficiency levels, but also reveals an interesting asymmetry in the effect: the bottom quintiles of the model population experience smaller declines relative to the top quintiles. Across the baseline calibration and experiment 3, the IRRs for the respective groups decline by roughly 18, 30, 39, 46 and 52 percent, which implies that the burden of population aging is lesser on the poorer households who actually benefit from social security. Given this fact, it appears intuitive that the optimal or welfare-maximizing response requires relatively small changes in the tax rate.

Why does population aging impose a lesser burden on the poorer households who benefit the most from the social security program? Examination of equation (25) reveals that the answer to this question lies in understanding how it affects household labor supply over the life cycle: a crucial determinant of the IRR. In Table 6, I report the expected labor supply over the life cycle (in efficiency units) for the different efficiency groups under the three experiments, with the tax rate held fixed at the baseline level. Two facts are clear from the table. First, households respond by increasing their

Table 6: The effect of population aging on the households' labor supply over the life cycle.

Experiment	$\varphi = 0.2$	$\varphi = 0.4$	$\varphi = 0.6$	$\varphi = 0.8$	$\varphi = 1$
Baseline	1.25	2.91	4.57	6.23	7.89
1	1.32	3.1	4.89	6.68	8.47
2	1.39	3.27	5.16	7.05	8.94
3	1.47	3.45	5.43	7.41	9.4

labor supply: expected labor supply over the life cycle is higher than the baseline for all the efficiency groups. Second, the labor supply responses are not symmetric: the households with higher efficiency experience larger increases than the households with lower efficiency. Across the baseline calibration and experiment 3, expected labor supply over the life cycle increases by 17.5 and 18.7 percent for the bottom two quintiles, and by 19, 19.1 and 19.2 percent for the top three quintiles of the model population. As the relatively wealthy households supply more labor, they take up a larger tax burden relative to the poorer households, which causes a larger decline in their IRRs.

The life-cycle labor supply responses documented in Table 6 are a combined effect of responses both along the intensive margin (i.e. hours worked) as well as the extensive margin (i.e. retirement or the age of exiting the labor force). In Table 7, I decompose these changes: Sub-table 7a documents the average weekly hours spent on market work between ages 25-55 for the different efficiency groups (the intensive margin), and Sub-table 7b documents the actual retirement ages (the extensive margin). Note that in both the sub-tables I report post-population aging equilibrium values with the tax rate held fixed at the initial baseline. Sub-table 7a shows that in general, all the efficiency groups experience an increase in their weekly work hours due to population aging (the only exception are the poorest households under the low-cost experiment). Across the baseline and experiment 3, average weekly hours spent on market work increases by roughly 1.2, 2.5, 2.9, 3.1 and 3.2 percent respectively. With respect to the extensive margin, Sub-table 7b shows that the retirement age also increases under population aging. Across the baseline and experiment 3, the age at which labor supply drops to zero for the respective efficiency groups increases by 5.4, 4.6, 4.3, 4.2 and 4.1 years. Note that even with the largest delay in retirement, the smallest increase in the weekly hours for the poorest households leads to the smallest increase in their expected labor supply over the life cycle.

What is the final impact of population aging and the optimal tax response on the equilibrium social security benefits? In Table 8, I report the equilibrium retirement benefits of the households surviving to the eligibility age (T_{r}) under the three demographic experiments, with the OASI tax rate set at the optimal levels identified in

Table 7: Decomposing the labor supply responses along the intensive and the extensive margins.

(a) The intensive margin: average weekly hours spent on market work between ages 25-55.

Experiment	$\varphi = 0.2$	$\varphi = 0.4$	$\varphi = 0.6$	$\varphi = 0.8$	$\varphi = 1$
Baseline	29.9	34.07	35.44	36.13	36.54
1	29.75	34.24	35.73	36.47	36.91
2	29.95	34.6	36.14	36.91	37.37
3	30.25	34.92	36.48	37.25	37.72

(b) The extensive margin: actual retirement age.

Experiment	$\varphi = 0.2$	$\varphi = 0.4$	$\varphi = 0.6$	$\varphi = 0.8$	$\varphi = 1$
Baseline	61.48	63.09	63.61	63.86	64.02
2	64.37	65.88	66.24	66.41	66.5
3	65.9	66.89	67.17	67.3	67.38
4	66.9	67.67	67.91	68.02	68.09

Table 4. The table shows that population aging always leads to a decline in social security benefits even in the presence of the optimal or welfare-maximizing tax response: benefits fall by roughly 18, 21 and 22 percent under the three experiments. However, it is important to note that these declines are significantly smaller than what would have occurred if the tax rate were held fixed at the baseline level. Social security benefits in post-population aging equilibria are about 28, 36 and 43 percent lower under the three demographic experiments with $\theta = 0.107$.

5 Conclusions

In the current research I ask what should be the optimal or welfare-maximizing OASI tax rate in the U.S. under the projected future demographics. I construct a heterogeneous - agent general equilibrium model of life-cycle consumption and labor supply, where the source of heterogeneity is a productivity or efficiency realization that occurs before the agents enter the model. In the model, an unfunded social security program provides partial insurance against the unfavorable efficiency realization by paying retirement benefits through a pro-poor rule. I first calibrate the benefit rule to match the

Table 8: Equilibrium social security benefits with the optimal tax response.

Experiment	$\varphi = 0.2$	$\varphi = 0.4$	$\varphi = 0.6$	$\varphi = 0.8$	$\varphi = 1$
Baseline	0.0171	0.0342	0.0513	0.0684	0.0855
1	0.014	0.0279	0.0419	0.0558	0.0698
2	0.0136	0.0271	0.0407	0.0543	0.0678
3	0.0133	0.0266	0.0399	0.0532	0.0665

degree of redistribution in the U.S. program, and then calibrate the model's efficiency distribution such that the current OASI tax rate in the U.S. is optimal under the current demographics. Then, I introduce empirically reasonable population projections from the 2009 OASDI Trustees Report into the calibrated model, and finally search for the tax rates that maximize social welfare under those projections. I find that the optimal tax rates under the projected future demographics in the U.S. are roughly 2 to 5 percentage points higher than the current rate. I also find that population aging has a smaller impact on the relatively poor households who benefit from social security, as wealthier households respond by supplying more labor and picking up a larger tax burden. Finally, the model also predicts that population aging and the optimal tax response may imply a decline in the projected retirement benefits.

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