Is the Social Security Crisis Really as Bad as We Think?*

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Abstract

Because they ignore the endogenous household-level and macroeconomic adjustments associated with longevity improvements, the actuarial projections of the Social Security Administration overestimate the Social Security crisis. I show that accounting for these adjustments, the decline needed in the future benefits to keep Social Security solvent is only about 63% of what the actuarial estimates predict. Households respond to the longevity improvements by working more hours and delaying retirement, which increases the aggregate labor supply, and by also saving more, which increases the aggregate capital stock and the wage rate. In general equilibrium, these effects lead to a natural expansion of the Social Security tax base.

JEL Classifications: E21, H55, J22

Keywords: Social Security; population aging; life-cycle consumption; labor supply; general equilibrium

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1 Introduction

Over the last few decades, mitigating the effect of longevity improvements on unfunded social security programs has been a major policy concern in the developed world. According to the 1998 World Health Report, by the year 2025, 26 countries will have an average life expectancy at birth of more than 80 years.[1] As is well known, these demographic developments will significantly worsen the health of unfunded public pension systems in these countries. In the U.S., life expectancy at birth is projected to increase to slightly over 81 years for males, and to roughly 85 years for females by 2075 (Shrestha, 2006). According to the 2011 Social Security Trustees Report, the current payroll tax rate for the Old-Age and Survivors Insurance (OASI) program is sufficient to pay only 77% of scheduled benefits in 2036, and only 74% of scheduled benefits in 2085.

One difficulty with the projections of the Social Security Administration (SSA) is that they are purely actuarial in nature, and are therefore subject to the Lucas critique.[2] It is easy to see that increased longevity will have a significant effect on the expenditure side of Social Security: households that expect to live longer will also collect benefits for longer, thereby increasing future Social Security spendings. With unchanged revenues, the only way to keep Social Security solvent in such a case would be to reduce the future benefit per retiree. The fact that only 77% and 74% of the scheduled benefits in 2036 and 2085 will be payable with the current contribution rate, implies that the benefit per retiree will have to be cut by 23-26% by the end of the century.

But the improvements in longevity will also affect future Social Security revenues, the tax base in particular. First, a higher life expectancy will directly increase labor supply, because there will be more workers alive at any age, and also because it may give households an incentive to work more hours and delay retirement. Second, if households risk out-living their assets, then a higher life expectancy will also induce higher saving, and therefore stimulate the aggregate capital stock. In general equilibrium, these effects will lead to a natural expansion of the Social Security tax base. The SSA's actuarial projections do make some assumptions about how the macroeconomic variables relevant for Social Security will change over time, but those assumed changes are completely exogenous. They overlook the endogenous household-level and macroeconomic adjustments to the improvements in longevity in their estimates of the future health of Social Security. And because these estimates are frequently used to legislate changes in Social Security laws, any bias in them will lead to incorrect policy.

In this paper, I compute the extent by which the SSA's actuarial projections overestimate the Social Security crisis in the U.S. To do this, I construct a heterogeneous-agent general-equilibrium model of life-cycle consumption and labor supply, where the source of heterogeneity is a permanent productivity shock realized before the agents enter the model. In the model, Social Security provides partial insurance against an unfavorable productivity shock by paying benefits though a progressive benefit-earnings rule.[3] Factor markets in the model are competitive, and firms maximize profit. I calibrate the model to match some key features of the U.S. economy, and then examine how a singular improvement in life expectancy affects the budget-balancing Social Security benefit. I find that accounting for

[1] The World Health Reports are published by the World Health Organization.

[2] See Ljungqvist (2008) for a discussion on the Lucas critique.

[3] Because there is mortality risk in the current model, Social Security can also be though of as providing longevity insurance. However, Guo et al. (2012) demonstrate that unfunded social security can improve welfare as an imperfect annuity only when the accidental bequests from the dying households are exogenous.

all the endogenous household-level and macroeconomic adjustments to the improvements in longevity, the percentage decline in the benefits required to keep Social Security solvent is significantly smaller than what the actuarial estimates predict. Therefore, using a model that satisfies the Lucas critique, I show that the Social Security crisis in the U.S. is really not as bad as we may have been led to believe.

In terms of the nature of the exercise undertaken, this paper is very similar in spirit to Chen and İmrohoroğlu (2012), who quantitatively examine the implications of different expenditure projections, such as those provided by the Congressional Budget Office (CBO) and The National Commission on Fiscal Responsibility and Reform, on the future debt-to-GDP ratio in the U.S. They demonstrate that the CBO's projections are likely to underestimate the future debt-to-GDP ratio, as they ignore the endogenous labor and capital responses to the changes in the marginal tax rates used in the projections. In this paper, I show that the SSA's projections are likely to overestimate the Social Security crisis in the U.S., because they ignore the very same endogenous responses to the improvements in longevity.

There are two channels through which a higher life expectancy leads to a natural expansion of the Social Security tax base. Depending on the demographic experiment, I find that labor supply goes up by roughly 4-15% from the baseline, both because workers remain alive for longer, and also because they work more hours per week and delay retirement. Capital stock also goes up, but by a significantly larger amount (10-39%) as households save more, both because they supply more labor and therefore earn higher income, and also because they have to smooth consumption over a longer expected lifespan. Together, these changes lead to a 2-7% increase in the equilibrium wage rate. Given that the Social Security tax base is simply the product of the wage rate and labor supply, these constitute a roughly 6-22% expansion in the future Social Security revenues. Once this expansion is accounted for, the budget-balancing decline required in the Social Security benefit is only 63% of what the actuarial estimates predict. I also find that the effect of labor supply on the tax base is more important than that of the wage rate. The actuarial projections end up overestimating the Social Security crisis by only 10% if we ignore adjustments in the wage rate, but by about 18% when we ignore how households adjust their labor supply over the life-cycle.

Economists have long emphasized the importance of studying Social Security reform using models that account for the endogenous general-equilibrium effects of aggregate shocks in an economy. For example, De Nardi et al. (1999) demonstrate that the SSA's projections about the future tax rates required to keep Social Security solvent in the U.S. may be overly optimistic, as they overlook the distortions imposed by those higher tax rates on household behavior. They show that higher taxes are likely to discourage labor supply and saving, which are likely to have a quantitatively important effect on the future income rate of the program. Jeske (2003) demonstrates that the privatization of Social Security can be beneficial for all future generations even in the presence of aggregate shocks, if the general-equilibrium effects of the privatization are accounted for. He shows that privatization of social security is likely to increase private saving and therefore the aggregate capital stock, which would lead to an improvement in welfare large enough to compensate against even large aggregate shocks. Therefore, the general finding is that the implications of Social Security reform can be markedly different depending on whether or not the associated general-equilibrium effects have been accounted for. Other studies that have used general-equilibrium models to examine alternative proposals to reform Social Security in the U.S. include Huang et al. (1997), Huggett and Ventura (1999), Conesa and Garriga (2008a,b), and Kitao (2012). The current paper can be thought of as complementary to these studies, as it demonstrates that general-equilibrium effects are quantitatively important not only in the context of evaluating Social Security reform policies, but also in the measurement of the crisis itself.[4]

The household-level decision margins that I account for in this paper are well-documented in the literature. From the perspective of an individual, labor supply over the life-cycle can be characterized by two margins: the extensive margin, which determines the fraction of lifetime spent in employment, and the intensive margin, which determines the hours of work supplied when employed. These margins have been found to be empirically relevant both in an aggregate sense, and also individually. For example, Wallenius (2009) finds that differences in social security account for 35-40% of the differences in aggregate hours between the U.S. and Belgium, France and Germany. Additionally, Prescott et al. (2009) note that about half of the differences in aggregate hours worked between the U.S. and continental Europe can be accounted for by the differences in the fraction of lifetime worked. However, as Rogerson and Wallenius (2009) demonstrate, it is necessary to account for both the intensive and the extensive margins to correctly estimate the elasticity of labor supply to tax and transfer programs. They show that large aggregate elasticities are consistent with small micro or life-cycle elasticities if both the extensive and intensive margins are accounted for.[5] Given the empirical relevance of both margins, in the current model I allow households to optimally choose the fraction of time spent in leisure (the intensive margin of labor supply), while also allowing it to equal unity if necessary (the extensive margin).

The rest of the paper is organized as follows. In Section 2, I describe in detail how the SSA measures the future health of Social Security. I introduce the general-equilibrium model in Section 3, and describe the baseline calibration in Section 4. In Section 5, I describe the demographic experiments, and also compute the actuarial estimates of the Social Security crisis from the model. I discuss the general-equilibrium estimates of the Social Security crisis in Section 6, and I measure the relative importance of the individual mechanisms at work in Section 7. Section 8 contains a sensitivity analysis of the baseline results, and Section 9 concludes.

2 The SSA's methodology

The Social Security Administration annually reports the financial health of the Old-Age and Survivors Insurance (OASI) and the Disability Insurance (DI) Trust Funds in the SSA Trustees Report. Actuarial status of the OASDI program is calculated both in the short-(10 years) and long-range (75 years), using specific definitions for the program income and cost rates. Also, both the short- and long-range estimates are presented under three alternative sets of assumptions: low-cost, intermediate, and high-cost. The intermediate assumptions represent the Board of Trustees' best estimate of the future course of the population and the economy, whereas the low-cost and the high-cost assumptions represent more optimistic and more pessimistic estimates, respectively. According the 2011 report, non-interest income in the OASDI program is projected to be sufficient to pay only about 77% of scheduled benefits in 2036, and 74% of scheduled benefits in 2085 based on the intermediate assumptions.

[4]General equilibrium effects have also been found to be quantitatively important in resolving other macroeconomic questions, such as the non-monotonicity of life-cycle consumption (Bullard and Feigenbaum, 2007; Feigenbaum, 2008) and the welfare-improving role of unfunded social security in a fully rational economy (İmrohoroğlu et al., 1995).

[5]Focusing only on the extensive margin of household labor supply, Ortiz (2009) finds that roughly 90% of the differences in the employment-to-population ratio at ages 60-64 across the OECD can be explained by the differences in the institutional features of social security.

The future financial status of the OASDI program depends on several key variables, such as mortality, average earnings, labor force participation rates, and inflation. In the Trustees' Report, the SSA makes specific assumptions about how the values of each of these variables change over time. For example, average life expectancy at birth in the U.S. is assumed to reach 81.3, 85, and 88.9 years by 2085 under the low-cost, intermediate, and high-cost assumptions. Also, between 2020 and 2085, average nominal U.S. earnings are assumed to grow at the rate of 3.6%, 4%, and 4.4% per annum respectively. Among other factors, these increases in earnings reflect trend increases of 0.1%, 0.0%, and -0.1% per annum in the average hours worked in the U.S.

Even though the Trustees Report accounts for how the macroeconomic variables relevant for Social Security change over time, their assumed changes are completely exogenous. For example, the labor force participation rate projections of the SSA reflect the trend effect of increases in life expectancy, higher assumed disability prevalence rates, and an increasing proportion of males who never marry. Improved life expectancy will trivially increase the labor force participation rate, simply because there will be more workers alive at every age. However, if households respond to the improved life expectancy by working more, then the labor force participation rate will increase even further. Also, if the households risk outliving their assets, then a higher life expectancy will also induce higher saving. To measure the combined effect of these changes on the future financial status of Social Security, one would need an equilibrium model in which all of the household-level and the macroeconomic adjustments are endogenous. In other words, one would need a model that satisfies the Lucas critique to correctly estimate the magnitude of the Social Security crisis in the U.S.

3 The model

Consider an overlapping generations economy with life-cycle permanent income households, where at each instant a new cohort is born and the oldest cohort dies. Cohort size grows at the rate of n per annum. Within each cohort there is income heterogeneity and the fraction of households of type i is f_i, where $\sum_i f_i = 1$. Maximum lifespan is T and households face a probability $Q(s)$ of surviving to age s. Therefore, total population at date t

$$P(t) = \sum_i f_i \sum_{s=0}^{T} N(t-s)Q(s) \tag{1}$$

grows at rate n over time, where $N(t-s)$ is the size of the cohort born at date $t-s$.

Over the life cycle, households accumulate a risk-free asset: physical capital. Private annuities markets are closed, and the assets of the deceased at each instant are uniformly distributed over the surviving population in the form of accidental bequests.[6,7] Households also earn labor income if they are working, and from age T_r onwards, they receive Social

[6]Assuming closed private annuities markets is standard in this line of literature, and is also empirically consistent because in reality very few people annuitize. This phenomenon is referred to as the "non-annuitization" puzzle, because a standard life-cycle model predicts that households ought to invest exclusively in annuities if they are fairly priced. Explanations behind this puzzle include existence of pre-annuitized wealth in retirees' portfolios, actuarially unfair prices, bequest motives, and uncertain health expenses. See, for example, studies such as Pashchenko (2010), Dushi and Webb (2004), Mitchell et al. (1999), Lockwood (2012), and Turra and Mitchell (2004).

[7]Hendricks (2002) finds that accidental or unintended bequests account for at least half of observed bequests in the U.S.

Security benefits that are positively linked to their work-life income. Firms operate competitively and produce output using capital, labor and a constant returns to scale technology. Finally, the government runs two programs: Social Security financed through a payroll tax (τ_{ss}), and a general tax and transfer program financed through an income tax (τ_y). There is also technological progress at the rate of g per annum.

3.1 Preferences

Period utility depends on both consumption (c) and the fraction of total time endowment enjoyed in leisure (l). It has the standard CIES form

$$u(c,l) = \begin{cases} \frac{\left(c^{\eta} l^{1-\eta}\right)^{1-\sigma}}{1-\sigma} & \text{if} \quad \sigma \neq 1 \\ \ln\left(c^{\eta} l^{1-\eta}\right) & \text{if} \quad \sigma = 1 \end{cases} \tag{2}$$

where η is the share of consumption, and σ is the inverse of intertemporal elasticity. Expected lifetime utility from the perspective of a household of type i born at date t is

$$U_i = \sum_{s=0}^{T} \beta^s Q(s) \, \frac{\left\{c_i(t+s,t)^{\eta} l_i(t+s,t)^{1-\eta}\right\}^{1-\sigma}}{1-\sigma} \tag{3}$$

where β is the discount factor. Also, since I define leisure as a fraction of the total time endowment, $0 \leq l(t+s,t) \leq 1$.

3.2 Income

A household of type i born at date t earns net of taxes wage income $(1-\tau_{ss}-\tau_y)(1-l_i(t+s,t))w(t+s)e(s)\varphi_i$ at every age s, where $w(t+s)$ is the wage rate, $e(s)$ is an age-dependent efficiency endowment and φ_i is a permanent productivity shock that occurs prior to birth. After age T_{r}, a household of type i receives Social Security benefits $b_i(t+s)$ until death, and every household receives a lump-sum welfare payment $\chi(t+s)$ every period of the life cycle. The surviving households also receive an accidental bequest $B(t+s)$ from the deceased every period.

3.3 Social Security and tax-and-transfer

In the model, Social Security provides partial insurance against an unfavorable permanent productivity shock through a progressive benefit-earnings rule. The benefit at date $t+s$ for a household with the productivity shock φ_i is $b_i(t+s)$, which is a concave function of past (work-life) income. The general tax-and-transfer program is also progressive: it collects a constant fraction of household earnings (which depends on the permanent productivity shock) and pays out a uniform benefit $\chi(t+s)$ to each surviving household. The government balances the budget for both of these programs.

3.4 Household optimization problem

A household of type i born at date t faces the following optimization problem

$$\max_{c_i,l_i} U_i = \sum_{s=0}^{T} \beta^s Q(s) \, \frac{\left\{c_i(t+s,t)^{\eta} l_i(t+s,t)^{1-\eta}\right\}^{1-\sigma}}{1-\sigma} \tag{4}$$

subject to

$$c_i(t+s,t) + k_i(t+s+1,t) = (1+r)k_i(t+s,t) + y_i(t+s,t) + B(t+s) + \chi(t+s) \quad (5)$$

$$y_i(t+s,t) = (1-\tau_{ss}-\tau_y)(1-l_i(t+s,t))w(t+s)e(s)\varphi_i + \Theta(s-T_{\mathrm{r}})b_i(t+s) \quad (6)$$

$$0 \leq l_i(t+s,t) \leq 1 \quad (7)$$

$$k_i(t,t) = k_i(t+T+1,t) = 0 \quad (8)$$

where

$$\Theta(x) = \begin{cases} 0 & x \leq 0 \\ 1 & x > 0 \end{cases}$$

is a step function.

3.5 Technology and factor prices

Output is produced using a Cobb-Douglas production function with inputs capital, labor and a stock of technology $A(t)$

$$Y(t) \;=\; K(t)^{\alpha}\left(A(t)L(t)\right)^{1-\alpha} \quad (9)$$

where $A(t) = A(0)(1+g)^t$, α is the share of capital in total income and $A(0)$ is the initial stock of technology. Firms face perfectly competitive factor markets, which implies

$$r = MP_K - \delta = \alpha\left[\frac{K(t)}{A(t)L(t)}\right]^{\alpha-1} - \delta \quad (10)$$

$$w(t) = MP_L = A(t)(1-\alpha)\left[\frac{K(t)}{A(t)L(t)}\right]^{\alpha} \quad (11)$$

where δ is the depreciation rate of physical capital and $w(t)$ is the wage rate at time t. In the steady-state, the wage rate grows at rate g per annum, and the rate of return r is constant.

3.6 Aggregation

Aggregate capital stock and labor supply are given by

$$K(t) = \sum_i f_i \sum_{s=0}^{T} N(t-s)Q(s)\, k_i(t,t-s-1) \quad (12)$$

$$L(t) = \sum_i f_i \sum_{s=0}^{T} N(t-s)Q(s)\, \{1-l_i(t,t-s)\}\, e(s)\varphi_i \quad (13)$$

Also, total accidental bequests paid to the surviving households must equal the total value of the assets left behind by the deceased households at each instant

$$\begin{aligned} B(t)P(t) \;&=\; (1+r)\left[\sum_i f_i \sum_{s=0}^{T}\{N(t-s)Q(s) - N(t-s-1)Q(s+1)\}\, k_i(t,t-s-1)\right] \\ &\quad - \sum_i f_i \sum_{s=0}^{T}\left(N(t-s+1) - N(t-s)\right)Q(s)k_i(t+1,t-s) \quad (14) \end{aligned}$$

Note that in a model with mortality risk and population growth, the number of households between two successive ages changes because of two reasons: only a fraction of each cohort survives to the following age, and over time cohorts get successively larger. The first term on the right-hand side of (14) gives the total assets left behind because of these two reasons. Therefore, to isolate the assets that are left behind purely because of mortality risk (i.e. by the households that die between ages s and $s+1$), the second term on the right-hand side of (14) reduces total assets by the part that is attributable only to population growth. In the absence of population growth, i.e. when $N(t-s) = N(t-s-1) = N$, (14) collapses to

$$B(t)P(t) = N \times (1+r)\left[\sum_i f_i \sum_{s=0}^{T} \{Q(s) - Q(s+1)\} k_i(t, t-s-1)\right] \tag{15}$$

which can be rewritten as

$$B(t)P(t) = N \times (1+r)\left[\sum_i f_i \sum_{s=0}^{T} h(s)Q(s)k_i(t, t-s-1)\right] \tag{16}$$

where $h(s) = -\left(Q(s+1) - Q(s)\right)/Q(s)$ is the hazard rate of dying between age s and $s+1$. It is easy to see that the right-hand side of (16) now only contains the assets left behind by the households that do not survive to the following ages.

Finally, the budget-balancing conditions for Social Security and the tax-and-transfer program are respectively

$$\sum_i f_i \sum_{s=0}^{T} N(t-s)Q(s)\tau_{ss}\left(1 - l_i(t, t-s)\right) w(t)e(s)\varphi_i = \sum_i f_i \sum_{s=0}^{T} N(t-s)Q(s)\Theta(s-T_r)b_i(t) \tag{17}$$

$$\sum_i f_i \sum_{s=0}^{T} N(t-s)Q(s)\tau_y\left(1 - l_i(t, t-s)\right) w(t)e(s)\varphi_i = \sum_i f_i \sum_{s=0}^{T} N(t-s)Q(s)\chi(t) \tag{18}$$

Total consumption and investment expenditures in this economy sum up to the total incomes earned by capital and labor:

$$C(t) + K(t+1) - K(t) = C(t) + (n+g+ng)K(t) = w(t)L(t) + rK(t) \tag{19}$$

3.7 Equilibrium

A competitive equilibrium in the current model can be characterized by a collection of

1. cross-sectional consumption allocations $\{c_i(t, t-s)\}_{s=0}^{T}$, asset allocations $\{k_i(t, t-s)\}_{s=0}^{T}$, and labor supply allocations $\{1 - l_i(t, t-s)\}_{s=0}^{T}$,
2. aggregate capital stock $K(t)$ and labor $A(t)L(t)$,
3. rate of return r and wage rate $w(t)$,
4. Social Security benefits $b_i(t)$ and welfare payments $\chi(t)$, and
5. an accidental bequest $B(t)$

that

1. solves the household optimization problem,
2. equilibrates the factor markets,
3. balances Social Security and the tax-and-transfer program budgets,
4. satisfies the bequest-balance condition (14), and
5. satisfies the steady-state requirements $c_i(t,t-s) = c_i(t+s,t)(1+g)^{-s}$, $k_i(t,t-s) = k_i(t+s,t)(1+g)^{-s}$, $B(t+s) = B(t)(1+g)^s$, $b_i(t+s) = b_i(t)(1+g)^s$ and $\chi(t+s) = \chi(t)(1+g)^s$.

The equilibrium computation methodology for this model is relatively straightforward. Also, since I focus only on steady-state analysis, I set $t = 0$ and normalize initial newborn cohort size and technology to $N(0) = A(0) = 1$.

4 Baseline calibration

I calibrate the baseline equilibrium of the model using empirical evidence from various sources. A population growth rate of $n = 1\%$ is consistent with the U.S. demographic history, and I set the rate of technological progress to $g = 1.56\%$, which is the trend growth rate of per-capita income in the postwar U.S. economy (Bullard and Feigenbaum, 2007). Households enter the model at actual age 25, which corresponds to the model age of zero. I obtain the baseline survival probabilities from Feigenbaum's (2008) sextic fit to the mortality data in Arias (2004), which is given by

$$\begin{aligned} \ln Q(s) &= -0.01943039 + \left(-3.055 \times 10^{-4}\right) s + \left(5.998 \times 10^{-6}\right) s^2 \\ &\quad + \left(-3.279 \times 10^{-6}\right) s^3 + \left(-3.055 \times 10^{-8}\right) s^4 + \left(3.188 \times 10^{-9}\right) s^5 \\ &\quad + \left(-5.199 \times 10^{-11}\right) s^6 \end{aligned} \tag{20}$$

where s is model age. The 2001 U.S. Life Tables in Arias (2004) are reported up to actual age 100, so I set the maximum model age to $T = 75$. The resulting survivor function is plotted in Figure 1. Under these survival probabilities, the model life expectancy at birth turns out to be 78.6 years, which is slightly higher than the current projection of 77.9 years by the Centers for Disease Control and Prevention (CDC).[8] This divergence is likely because households in the model survive to age 25 with certainty (as the model age of zero corresponds to the actual age of 25), whereas in the real world they survive to age 25 with roughly 98% probability.

I set the model benefit eligibility age to $T_{\mathrm{r}} = 41$, which corresponds to the current actual full retirement age of 66 in the U.S. Following Conesa and Garriga (2008a), I parameterize the efficiency endowment profile $e(s)$ using data from Hansen (1993). However, as it is well-known, efficiency measured from wage data suffers from sample selection bias, especially at the later ages when a large number of households begin to retire. For this reason, I fit a quartic polynomial to the efficiency data in Hansen (1993) only for ages 25-65, which gives

$$\begin{aligned} \ln e(s) &= -3.273 \times 10^{-5} + \left(3.7484 \times 10^{-2}\right) s + \left(-1.7541 \times 10^{-3}\right) s^2 \\ &\quad + \left(3.4625 \times 10^{-5}\right) s^3 + \left(-2.7949 \times 10^{-7}\right) s^4 \end{aligned} \tag{21}$$

[8] See http://www.cdc.gov/nchs/fastats/lifexpec.htm.

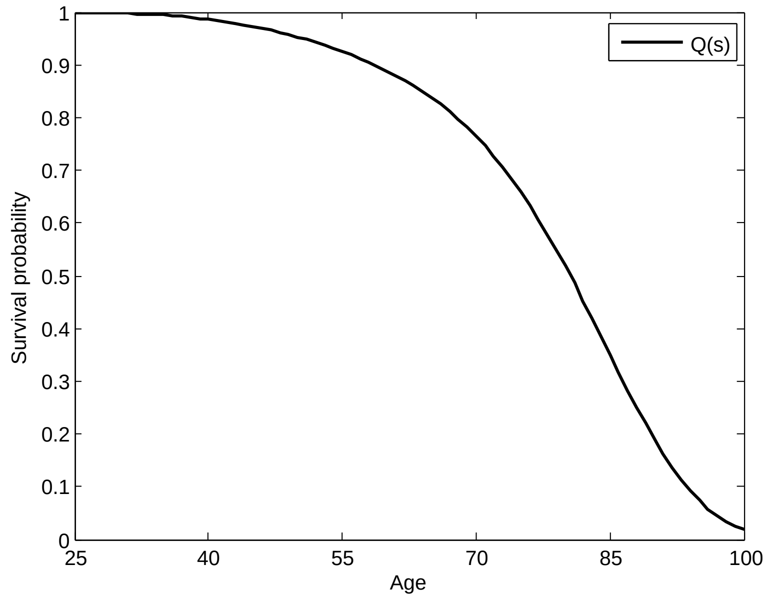

Figure 1: Survival probabilities from Feigenbaum's (2008) sextic fit to the mortality data in Arias (2004).

where s is model age and $s \leq 40$. Beyond actual age 65 (i.e. for $s > 40$), I use the following quadratic function

$$\ln e(s) = -f_0 - f_1 s - 0.01 s^2 \tag{22}$$

and parameterize f_0 and f_1 such that $e(s)$ is continuous and once differentiable at age $s = 40$.[9] Note that the coefficient of 0.01 on the squared term in (22) ensures that households do not continue to work beyond age 70.[10] The resulting efficiency endowment profile is plotted in Figure 2. I set the Social Security payroll tax rate to $\tau_{ss} = 0.106$, which is the combined OASI contribution rate in the U.S.

Social welfare payments in the U.S. are made in various forms. Payments from the OASI, Medicare and the Supplemental Security Income (SSI) programs are conditional on retirement, whereas payments from programs such as Food Stamps, hospital and medical care (excluding Medicare) and housing are not. The OASI benefit annuity in the U.S. is a concave (piecewise linear) function of work-life income. The Social Security Administration measures what is known as the Average Indexed Monthly Earnings (AIME) for every covered individual, and then replaces a fraction of the AIME. Depending on how large or small the AIME for an individual is relative to the average wage in the economy, the fraction that is replaced gets adjusted. For example, in 2001 the OASI benefit annuity in the U.S. was 90% of the AIME for the first \$561, 32% of the next \$3381, and 15% of the remaining upto the maximum creditable earnings. As shown by Huggett and Ventura (1999), these dollar amounts come out to be 20%, 124% and 247% of the average wage in the economy. These

[9]The values that satisfy these conditions are $f_0 = 15.4789$ and $f_1 = -0.7918$.

[10]The labor force participation rates for ages 70 and above in the U.S. are slightly over 10% from CPS data.

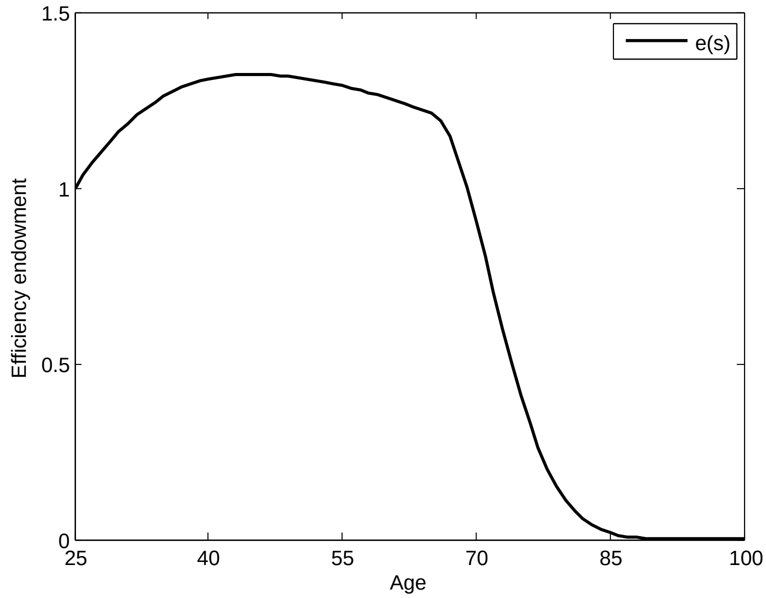

Figure 2: Efficiency endowment profile.

f_1	f_2	f_3	f_4	f_5
0.1872	0.1991	0.2128	0.196	0.2049

Table 1: Population shares of the five income quintiles.

percentage amounts are referred to as the "bend points" of the benefit rule, and I take them directly to the model. Note that the progressivity in the benefit rule is captured by the fact that the replacement rate is decreasing in the AIME (see Figure 3).

The historically observed value of capital's share in total income in U.S. ranges between 30-40%, so I set $\alpha = 0.35$. Finally, I view a household of type i in the model as representative of all the households in the i^{th} income quintile in the data. Therefore, to calibrate the f_i's, I compute the respective population shares of the five income quintiles using the U.S. income distribution in 2002 (see Table 1).

Once all the observable parameters have been assigned empirically reasonable values, the following unobservables remain to be calibrated:

1. the preference parameters σ (IEIS), β (discount factor) and η (share of consumption in period utility),
2. the permanent productivity shock (φ_i),
3. the income tax rate (τ_y), and
4. the depreciation rate (δ).

To calibrate the preference parameters and the depreciation rate, I use the following macroeconomic targets:

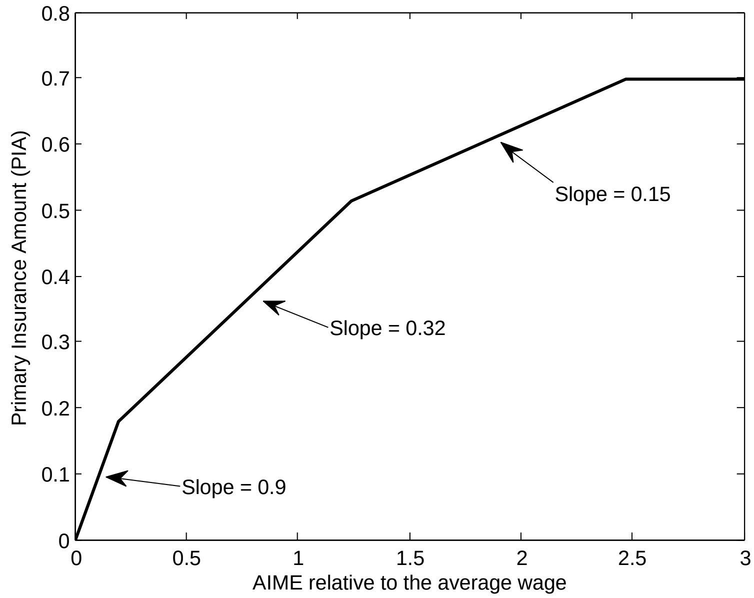

Figure 3: Benefit formula in the U.S.

1. a capital-output ratio of 3.0,
2. a ratio of aggregate consumption expenditure to income of 70%, and
3. an average of 34 hours per week spent on market work between ages 25-55.[11,12]

To calibrate the permanent productivity shock, I use data on measured income heterogeneity in the U.S. reported by The Urban Institute (2002). They report the distribution of AIME in successive birth cohorts for both men and women for the years 1926-1965. Earnings are reported both up to the actual taxable maximum, and also using a "less-censored" approach based on a uniform taxable maximum (indexed to the average wage in the economy) for all the birth cohorts. Using the "less-censored" earnings data, the ratio of the AIME of the bottom quintile to that of the top quintile for all eligible workers (averaging across all the birth cohorts and also men and women) turns out to be roughly 32%. I normalize the productivity shock for the top income quintile in the model to $\varphi_5 = 1$, and then calibrate the shock for the bottom income quintile (φ_1) such that the ratio of their model

[11] A capital-output ratio target of 3.0 and a consumption expenditure to income target of 70% implies that in the steady state, the depreciation rate is equal to $\delta = 0.0744$.

[12] Note that here three preference parameters are being calibrated using two macroeconomic targets, which implies that they are not being identified separately. Even though adopting life-cycle consumption data as an additional target breaks this observational equivalence, it turns out that the current model does a poor job of matching that target. See Section 8 for a more detailed discussion on this problem, and also for a sensitivity analysis of the baseline results with respect to the preference parameters.

σ	β	η	φ_1	τ_y
4	1.0165	0.2172	0.41	0.189

Table 2: Unobservable parameter values under the baseline calibration.

	Target	Model
Capital-output ratio	3.00	3.01
Consumption expenditure to income ratio	0.7	0.699
Avg. hours of market work per week between ages 25-55	34	34.08
$AIME_1/AIME_5$	0.32	0.323
$\left\{\sum_i f_i \sum_{s=0}^T N(t-s)Q(s)\chi(t)\right\}/Y(t)$	0.1228	0.1229

Table 3: Model performance under the baseline calibration.

AIMEs is 32%.[13] I obtain the productivity shocks for the middle income quintiles using linear interpolation.

Finally, to calibrate the income tax rate (τ_y), I use data on social welfare expenditures in the U.S.[14] According to the SSA, total social welfare spending in the U.S. is roughly 21% of GDP. Deducting OASI, Disability, Medicare, Railroad Retirement benefits, and Public Employee Retirement benefits, the remaining social welfare spending turns out to be 12.28% of GDP. This includes items such as food stamps, hospital and medical care (excluding Medicare), veterans programs, education, public housing, and several other social welfare services and benefits. I calibrate the income tax rate such that total payments from the tax-and-transfer program in the model at date t, which is equal to $\sum_i f_i \sum_{s=0}^T N(t-s)Q(s)\chi(t)$, matches 12.28% of GDP.

The unobservable parameter values under which the model reasonably matches the above targets are reported in Table 2. Note that a discount factor larger than unity implies a negative discount rate[15], which is common in the macro-calibration literature (Huggett, 1996; Bullard and Feigenbaum, 2007; Feigenbaum, 2008) as well as the quantitative public finance literature (Huggett and Ventura, 1999; Conesa and Garriga, 2008a; Nishiyama and Smetters, 2005). Also, note that with leisure in period utility, the relevant inverse elasticity for consumption is $\sigma^c = 1+\eta(\sigma-1) = 1.65$, which lies within the range frequently encountered in the literature. The model-generated values for the targets under the baseline calibration are reported in Table 3. The cross-sectional means of consumption and labor hours are reported in Figures 4 and 5.

Different realizations of the permanent productivity shock lead to significant differences in the present value of income over the life-cycle across the quintiles. In the baseline calibration, the present value of income over the life-cycle for households in the bottom quintile

[13] I compute the model AIMEs using the formula

$$AIME_i(t) = \frac{1}{T_{l_i=1}} \left\{ \sum_{s=0}^{T_{l_i=1}} \{1 - l_i(t, t-s)\} w(t)e(s)\varphi_i \right\}$$

Note that $T_{l_i=1}$ is the retirement age of households with efficiency φ_i, or the age at which labor supply drops to zero. Note that similar to the SSA's calculations, I index past wages to date t in computing the AIME.

[14] See http://www.socialsecurity.gov/policy/docs/progdesc/sspus/appeni.pdf.

[15] Here, $\rho = \frac{1-\beta}{\beta} = -0.0162$.

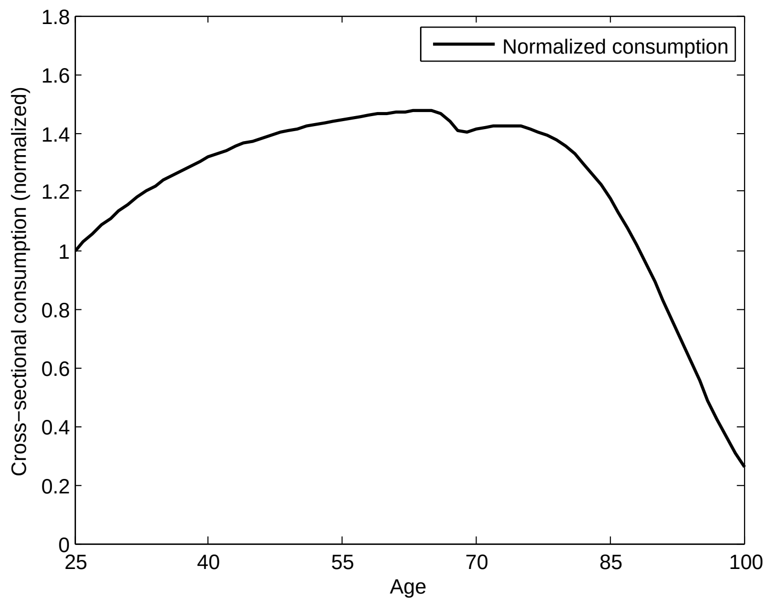

Figure 4: Baseline cross-sectional mean of consumption (normalized by consumption at age 25).

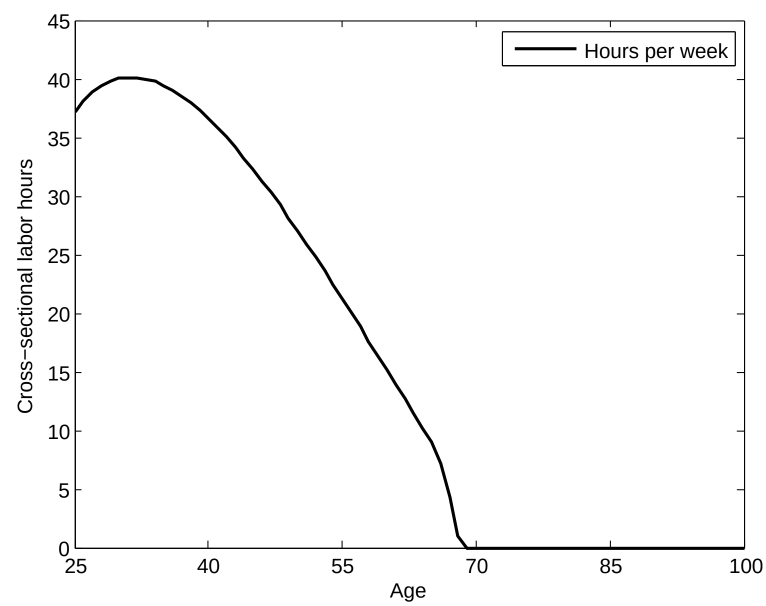

Figure 5: Baseline cross-sectional mean of labor hours per week.

Experiment	γ	μ	Life expectancy (years)
Low-cost (A)	1×10^{-4}	1.5774	81.3
Intermediate (B)	7×10^{-5}	1.8867	85
High-cost (C)	3×10^{-5}	2.2188	88.9

Table 4: The demographic experiments.

Projection	% change (from baseline)
A	-11.54
B	-24.61
C	-35.49

Table 5: Percentage changes in the Social Security benefit under Case 1.

is about 57% lesser than that of the households in the top quintile. A part of this is attributable to the fact that on the average, a household with the lowest possible realization of the productivity shock supplies roughly 30 hours per week in market work between ages 25-55 and retires at age 67, whereas a household with the highest possible productivity shock supplies roughly 37 hours per week, while retiring at age 69.

5 The longevity improvements

A straightforward way to incorporate a one-time improvement in longevity in the current model is to augment the baseline survival probabilities with an age-specific increment of the form

$$\mathrm{d}\, Q(s) = \gamma s^{\mu} \tag{23}$$

where s is model age, and γ and μ are positive constants that can be chosen to match specific life expectancy targets. Note that these age-specific increments are consistent with the fact that old-age survivorship in the U.S. has increased at a faster rate in the later half of the twentieth century, making the population survival curve more rectangular (Arias, 2004).

I choose values for γ and μ such that model life expectancy under the augmented survival probabilities matches the 2011 Trustees Report's average period life expectancy projections for the year 2085 under the low-cost, intermediate and high-cost assumptions. I define the specific demographic experiments in Table 4.[16] The survivor functions corresponding to the three experiments are compared to the baseline in Figure 6.

What are the actuarial estimates of the Social Security crisis under these projected survival probabilities? In Table 5, I report the percentage change in the Social Security benefit required to balance the program's budget under each demographic experiment using the SSA's methodology, i.e. without accounting for the endogenous household-level and macroeconomic adjustments (Case 1).[17] It clear from the table that the model does a fairly accurate job of matching the SSA's projections of the Social Security crisis in 2085 using their methodology. As mentioned earlier, according the 2011 Trustees' Report, non-interest income in the OASDI program is projected to be sufficient to pay roughly 74% of scheduled

[16] I hold the maximum lifespan unchanged at $T = 75$ under all the three experiments.

[17] In this case, economywide productivity in the model grows at the rate of 1.56% per annum, average hours worked grow at the rate of 0.0% per annum, and the labor force participation rate is constant. In the SSA's projections, economywide productivity grows at the rate of 1.7% per annum, average hours worked grow at the rate of 0.0% per annum, and the labor force participation rate is roughly constant.

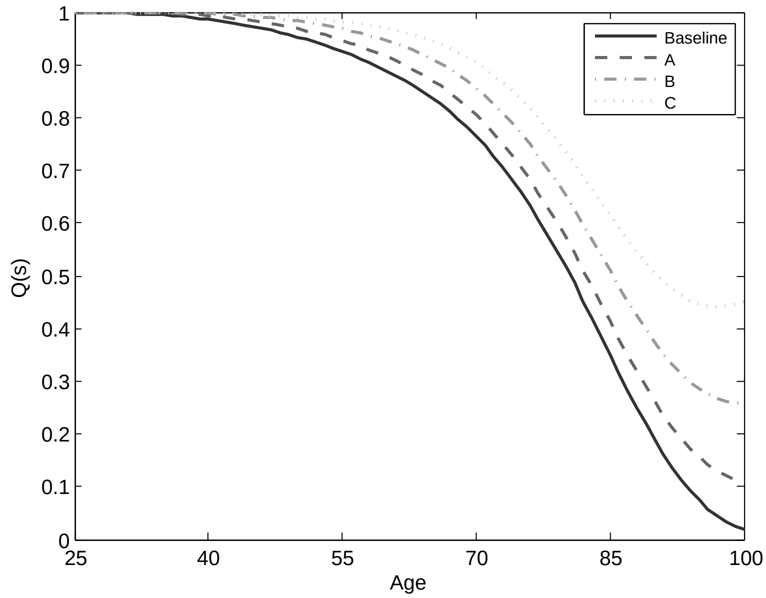

Figure 6: Baseline and the projected survival probabilities.

	Baseline	A	B	C
Avg. hours of market work per week between ages 25-55	34.1	34.4	35.1	35.8
Avg. retirement age	68.4	69	69.6	70.2
Labor	5.4	5.6	5.9	6.2
Capital	29.4	32.3	36.4	40.9
Output	9.8	10.4	11.2	12.0
Capital-output ratio	3.01	3.12	3.26	3.40

Table 6: Macroeconomic variables in the post-experiment equilibria.

benefits in 2085 under the intermediate assumptions. This implies a 26% decline in benefits, which is very close to the roughly 25% decline predicted by the model under experiment B for Case 1.

6 The general-equilibrium estimates

How does accounting for the endogenous household-level and macroeconomic adjustments affect the measured extent of the Social Security crisis? To answer this question, I incorporate the projected survival probabilities A, B, and C into the baseline model, but this time compute the budget-balancing change in the Social Security benefit (from the baseline) in general equilibrium (Case 2). For each experiment, I report the equilibrium values of the relevant macroeconomic variables in Table 6. It is clear from the table that the longevity improvements lead to an increase in the labor supply both along the extensive and the intensive margins. Between the baseline and experiment C, average cross-sectional hours per

Projection	φ_1	φ_2	φ_3	φ_4	φ_5	Average % change
A	-8.04	-7.87	-6.35	-6.69	-7.42	-7.28
B	-16.6	-15.4	-15.2	-15.3	-15.2	-15.5
C	-22.7	-22.7	-21.5	-22.6	-22.6	-22.4

Table 7: Percentage changes in the Social Security benefit under Case 2.

Projection	Case 1	Case 2	Case 2 relative to Case 1
A	-11.5%	-7.28%	63.0%
B	-24.6%	-15.5%	63.2%
C	-35.5%	-22.4%	63.2%

Table 8: Percentage changes in the Social Security benefit, Case 2 relative to Case 1.

week increases from 34 to almost 36, and the average retirement age increases from about 68 to slightly over 70. Capital stock and output also increase, but the increases in capital are proportionally larger, because of which the capital-output ratio increases between the baseline and experiment C.

The importance of accounting for the household-level and macroeconomic adjustments becomes clear once we compare the changes in the Social Security benefit under Case 2 with those under Case 1. Note that accounting for these adjustments introduces a complication: households with different realizations of the productivity shock respond differently along both the labor supply and saving margins. Since the benefit depends on past income, and therefore life-cycle labor supply, households with different productivity realizations experience different percentage changes in their benefits. Therefore, I also compute for Case 2 the average percentage change in the benefit (from the baseline) across all the productivity realizations. Both the individual and the average percentage changes under Case 2 are reported in Table 7. Comparing Table 7 to Table 5 shows that in general equilibrium (Case 2), the declines in the Social Security benefit are about 63% of what the actuarial estimates predict (Case 1) (see Table 8). In other words, ignoring the endogenous adjustments in the Social Security tax base that occur with the longevity improvements translates into a roughly 37% overestimation of the crisis itself.

Using the definition of aggregate labor supply from (13), we can rewrite the Social Security budget-balancing condition (17) as

$$\tau_{ss} w(t) L(t) = \sum_i f_i \sum_{s=0}^{T} N(t-s) Q(s) \Theta(s - T_r) b_i(t) \tag{24}$$

The left-hand side of equation (24) shows that the Social Security tax base is nothing but the product of the wage rate and the aggregate labor supply. Changes in both of these variables are endogenous under Case 2, but not under Case 1. In general equilibrium, increased hours per week and delayed retirement from households increases the aggregate labor supply, and higher saving increases the aggregate capital stock, and therefore the equilibrium wage rate. As Table 8 demonstrates, these endogenous adjustments are quantitatively important.

How do the aggregate labor supply and the wage rate individually adjust to the improvements in longevity? Figure 7 plots the cross-sectional mean of hours per week for the baseline calibration, and also for the experiments A, B and C. It is clear from the figure that there is actually a decline in the hours per week prior to about age 37 under all the

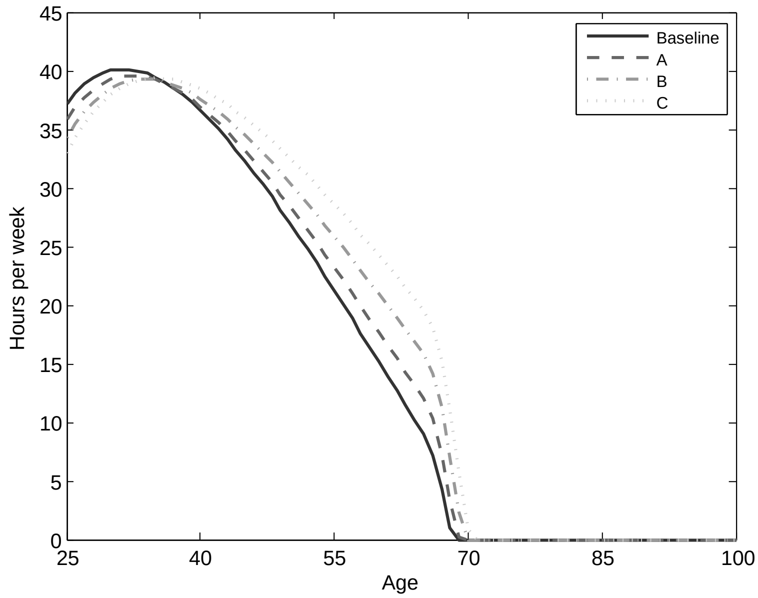

Figure 7: Cross-sectional means of hours per week, baseline and post-experiment.

three experiments. After age 37 until retirement, hours per week are always higher than the baseline. Averaged across all ages, cross-sectional labor time per week go up by roughly 35, 88, and 146 minutes respectively, and retirement is delayed by almost two years. Because household efficiency also peaks between ages 35-55, these reallocations in labor hours increase the aggregate labor supply by 3.9%, 9.4%, and about 15% respectively from the baseline, which is more than what can be accounted for by the hours alone.

If the capital stock were held fixed at the baseline level, these increases in the aggregate labor supply would reduce the equilibrium wage rate by 1.3%, 3.1%, and 4.8% respectively. Taken together, these changes would lead to an expansion of 2.6%, 6.3%, and 10.3% in the Social Security tax base. However, the capital stock also increases, as the households save more because of two reasons: they earn higher incomes due to the increased labor supply, and also because they have to smooth consumption over a longer expected lifespan. Averaging across all ages, cross-sectional mean asset holding increases from the baseline by about 22%, 48%, and 71% respectively. These changes in asset holding translate into a 9.9%, 23.8%, and 39.3% increase in the aggregate capital stock, and about 2%, 4.5%, and 6.9% increase in the equilibrium wage rate. Accounting for all of these adjustments, the Social Security tax base increases by about 5.9%, 13.8%, and 22% respectively under the three experiments.

7 Partial adjustments

As Table 8 demonstrates, the endogenous household-level and macroeconomic adjustments have a quantitatively important effect on the measured extent of the Social Security crisis. These adjustments come from equilibrium changes in both the wage rate and the aggregate

Projection	Case 1	Case 2	Case 3	Case 4
A	-11.5%	-7.28%	-9.19%	-10.8%
B	-24.6%	-15.5%	-20.0%	-22.1%
C	-35.5%	-22.4%	-29.3%	-31.0%

Table 9: Percentage changes in the Social Security benefit under Cases 3 and 4.

labor supply. How important are these elements individually? To examine this, I compute the budget-balancing change in the Social Security benefit under each demographic experiment, but now allow for only partial adjustments in the model economy. The specific experiments are as follows. First, I hold the weekly labor hours fixed at the initial baseline, and allow household saving and the wage rate be endogenous (Case 3), and second, I hold the wage rate fixed at the initial baseline, but allow only the household labor supply be endogenous (Case 4). Note that in both of the above cases, the government budgets (Social Security and the general tax-and-transfer program) are balanced, and the bequest-balance condition is also satisfied. The percentage changes in the benefit, averaged across the different productivity types, are reported in Table 9.

It is clear from Table 9 that both under Cases 3 and 4, the declines in the Social Security benefit are larger than Case 2, but smaller than Case 1. In fact, when I let only household saving and the wage rate be endogenous (Case 3), the decline in the benefit is about 82% of what the actuarial estimates predict (Case 1). On the other hand, when I let only the household labor hours be endogenous and hold the wage rate fixed at the baseline level (Case 4), the decline in the benefit is roughly 90% of what the actuarial estimates predict. In other words, the extent by which the actuarial projections overestimate the Social Security crisis is about 10% when we ignore adjustments in the wage rate, but is larger (about 18%) when we ignore how households adjust their labor supply over the life-cycle in response to a higher life expectancy.

Under Case 3, aggregate labor supply increases by only 0.97%, 1.8%, and 2.3% for the three experiments, which only reflect improvements in survivorship. Compared to this, the increases in aggregate labor supply under Case 4 are much larger: about 2.0%, 5.2%, and 9.5% respectively. This is because under Case 4, cross-sectional labor supply increases by about 6, 27, and 64 minutes per week (averaged across all ages), even though there is no change in the retirement ages (See Figure 8).

8 Sensitivity analysis

In a life-cycle consumption model with uninsurable mortality risk, the evolution of consumption and leisure over the work life are governed by the following two equations:[18]

$$\frac{c(t+s+1,t)}{c(t+s,t)} = \left(\frac{(1+g)e(s+1)}{e(s)}\right)^{\frac{(1-\eta)(\sigma-1)}{\sigma}} \left(\frac{Q(s+1)}{Q(s)}\beta(1+r)\right)^{\frac{1}{\sigma}} \tag{25}$$

$$\frac{l(t+s+1,t)}{l(t+s,t)} = \left(\frac{(1+g)e(s+1)}{e(s)}\right)^{\frac{\eta(1-\sigma)-1}{\sigma}} \left(\frac{Q(s+1)}{Q(s)}\beta(1+r)\right)^{\frac{1}{\sigma}}. \tag{26}$$

[18]Note that the evolution of consumption and leisure over the life-cycle does not depend on household type. The permanent productivity shock only has a wealth effect, in the sense that it only affects the life-cycle budget constraint.

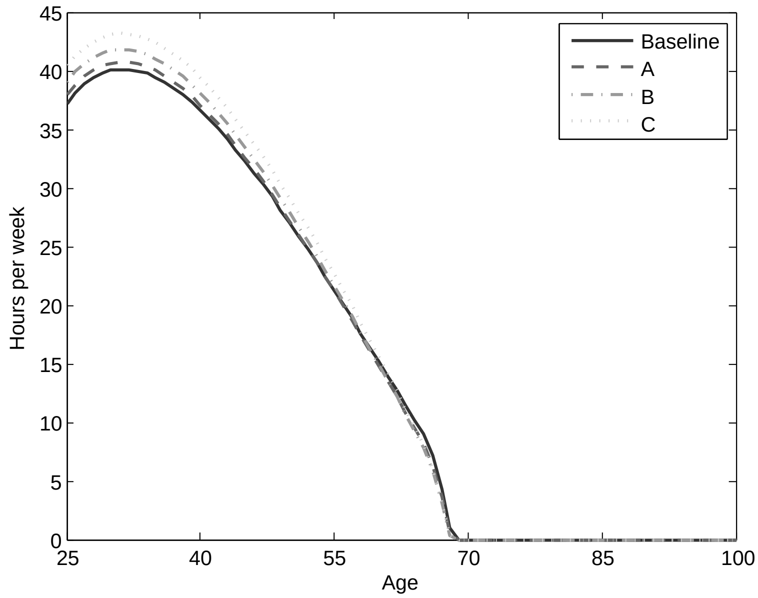

Figure 8: Cross-sectional means of hours per week under Case 4.

It is easy to see that with a given endowment profile $e(s)$, the target for average hours of market work per week helps pin down the share of consumption in period utility (η). Then, the inverse elasticity (σ) and the discount factor (β) can be separately identified using the capital-output ratio target and data on life-cycle consumption.

However, as it is well-known, no fully rational model can replicate the empirical life-cycle consumption profile in Gourinchas and Parker (2002) in a general equilibrium with Social Security.[19] This is because Social Security reduces private saving and therefore the aggregate capital stock, which leads to significantly higher interest rates in general equilibrium. A higher interest rate causes consumption to increase much more rapidly in early life, because of which the peaks in life cycle consumption in the model are too large and occur much later than what is found in data. For example, cross-sectional mean consumption in the baseline calibration peaks at roughly age 65, with a ratio of peak to initial consumption of about 1.5. The empirical consumption profile in Gourinchas and Parker (2002) peaks much earlier roughly at age 50, and is much flatter with a peak-to-initial consumption ratio of about 1.1. Because of the inability of the current model to match the empirical consumption profile, life-cycle consumption data from Gourinchas and Parker (2002) are not used as a target in the baseline calibration. As a consequence, the preference parameters σ and β are not separately identified in the baseline calibration.

This is problematic in the current context, as the quantitative importance of the general-equilibrium effects depends on the values of the unobservable preference parameters that are used to calibrate the model. How robust are the baseline results, given that multiple sets of unobservable parameters are consistent with the same macroeconomic targets? To examine

[19]Bullard and Feigenbaum (2007) find a calibration that reasonably matches the empirical consumption profile, but using a model without Social Security.

σ	σ^c	β	η	φ_1	τ_y
1	1	0.9888	0.2082	0.38	0.189

Table 10: Unobservable parameter values under the baseline calibration with $\sigma = 1$.

	Target	Model
Capital-output ratio	3.0	3.0
Consumption expenditure to income ratio	0.7	0.699
Avg. hours of market work per week between ages 25-55	34	33.9
$AIME_1/AIME_5$	0.32	0.319
$\left\{\sum_i f_i \sum_{s=0}^{T} N(t-s)Q(s)\chi(t)\right\}/Y(t)$	0.1228	0.1229

Table 11: Model performance under the baseline calibration with $\sigma = 1$.

this, I re-calibrate the baseline model with $\sigma = 1$ and $\sigma = 6$, and then re-compute the budget-balancing Social Security benefit under each of the three demographic experiments A, B, and C in general equilibrium.

Let us first consider the case of $\sigma = 1$ or separable utility. Tables 10 contains the values of unobservable parameters, and Table 11 demonstrates how the model matches the macroeconomic targets under these parameter values.[20] Also, Figures 9 and 10 respectively plot the baseline normalized cross-sectional consumption and labor hours profiles. Because

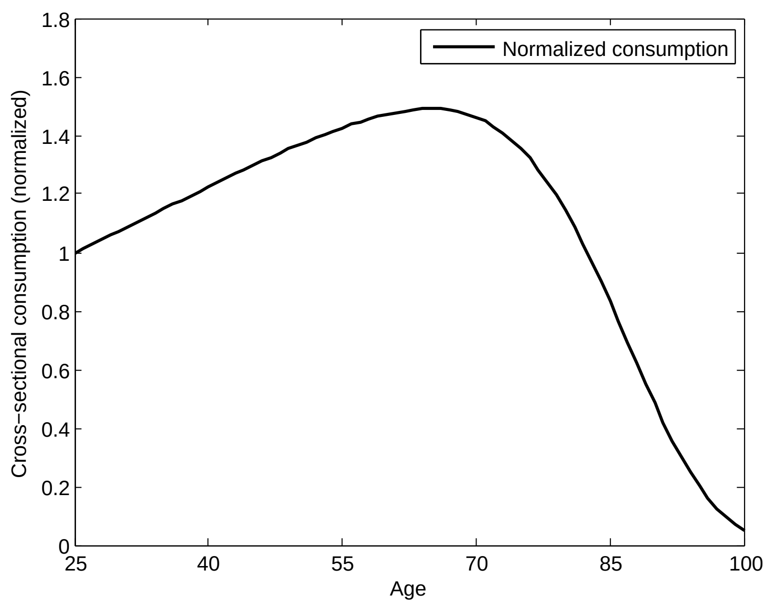

Figure 9: Cross-sectional mean of consumption (normalized by consumption at age 25) with $\sigma = 1$.

[20]Note that with separable utility, the intertemporal elasticity of consumption is identical to that of the consumption-leisure bundle.

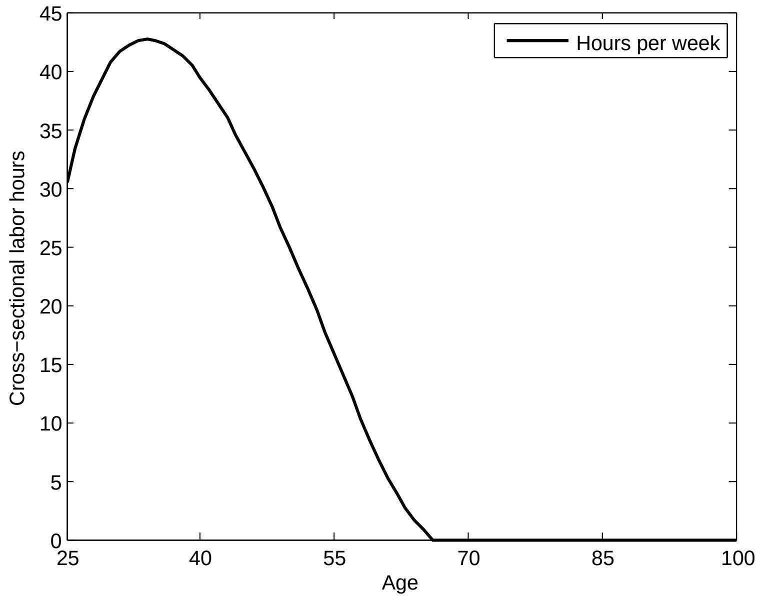

Figure 10: Cross-sectional mean of hours per week with $\sigma = 1$.

Projection	Case 1	Case 2	Case 2 relative to Case 1
A	-11.7%	-9.0%	77.0%
B	-24.9%	-18.2%	73.2%
C	-35.8%	-25.9%	72.2%

Table 12: Percentage changes in the Social Security benefit under Cases 1 and 2 with $\sigma = 1$.

utility is separable in consumption and leisure with $\sigma = 1$, there is no drop in consumption at the date of retirement (about age 66) in Figure 9. Also, the consumption profile peaks at about age 65, much later than what is found in data, with a ratio of peak-to-initial consumption of 1.5. How rapidly leisure changes between two successive ages over the work life depends on how rapidly the efficiency endowment $e(s)$, and also survivorship $Q(s)$, changes between those two ages (see equation (26)). The effect of the efficiency endowment is inversely proportional to the intertemporal elasticity σ^{-1}, whereas the effect of mortality is directly proportional to it. However, the consumption share parameter η limits the effect of the endowment profile on the evolution of leisure. Consequently, for a smaller $\sigma-$value (i.e. a higher elasticity) labor hours peak slightly later in life, because survivorship does not begin to decline sharply until later ages (see Figure 10), even though retirement occurs slightly earlier.

The effect of the longevity improvements on the budget-balancing Social Security benefit is reported in Table 12. As before, there are three demographic experiments A, B, and C, and the effect of each is reported both under Case 1 (i.e. using the SSA's methodology) and Case 2 (i.e. allowing the endogenous general-equilibrium adjustments). Also, note that the percentage changes are averaged across households with different productivity types for Case 2. Table 12 shows that even with $\sigma = 1$, the actuarial projections overestimate the

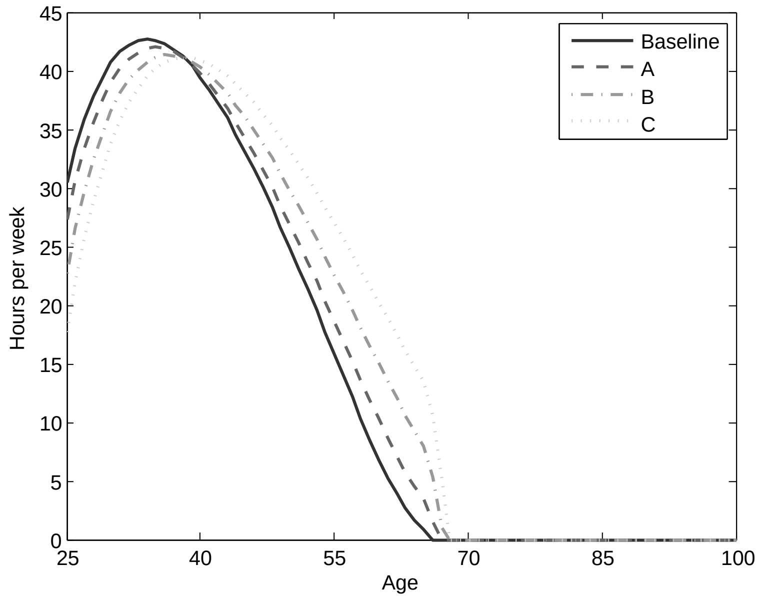

Figure 11: Cross-sectional means of hours per week, baseline and post-experiment, with $\sigma = 1$.

magnitude of the Social Security crisis, although by a slightly lesser extent. Across the three experiments, the percentage decline in the budget-balancing Social Security benefit under Case 2 is 72-77% of that under Case 1.

The percentage increases in labor supply under Case 2 are slightly smaller with $\sigma = 1$: roughly 3.5%, 8.4%, and about 14% respectively. Figure 11 reports the cross-sectional means of weekly hours for the three experiments, along with the baseline hours for $\sigma = 1$. Averaged across all ages, weekly labor supply increases from the baseline by roughly 30, 77, and 133 minutes: less than the increases with $\sigma = 4$. Also, higher life expectancy in this case leads to the hours per week declining from the baseline prior to age 39, and increasing thereafter. Note that this is slightly later than the cross-over age of 36 under $\sigma = 4$, and because the endowment profile is still increasing between ages 36 and 39, the work hours lost during these ages prevent labor supply from increasing by as much as it does with $\sigma = 4$.

The smaller increases in labor supply also lead to smaller increases in household saving under $\sigma = 1$. Across the three experiments, cross-sectional mean asset holding increases by roughly 26%, 59%, and 90% respectively, which are larger than the increases under $\sigma = 4$, but there is a key difference in how the asset holdings are distributed by age: asset holdings after age 60 are significantly smaller under $\sigma = 1$. Because the improvements in survivorship occur mostly at late ages, the aggregate capital stock increases by a smaller percentage under $\sigma = 1$: about 6.6%, 16.1%, and 26.7% respectively, in spite of the fact that the average asset holding increases by more.

Because the capital stock increases by a smaller percentage under $\sigma = 1$, the changes in the equilibrium wage rate are also smaller: 1%, 2.4%, and 3.8% for the three experiments. Taken together, these changes lead to a 4.5%, 10.8%, and 17.7% increase in the Social Security tax base under $\sigma = 1$. Given that these are somewhat smaller than the increases

σ	σ^c	β	η	φ_1	τ_y
6	2.11	1.0333	0.2211	0.41	0.189

Table 13: Unobservable parameter values under the baseline calibration with $\sigma = 6$.

	Target	Model
Capital-output ratio	3.0	2.99
Consumption expenditure to income ratio	0.7	0.7
Avg. hours of market work per week between ages 25-55	34	34.2
$AIME_1/AIME_5$	0.32	0.322
$\left\{\sum_i f_i \sum_{s=0}^{T} N(t-s)Q(s)\chi(t)\right\}/Y(t)$	0.1228	0.1228

Table 14: Model performance under the baseline calibration with $\sigma = 6$.

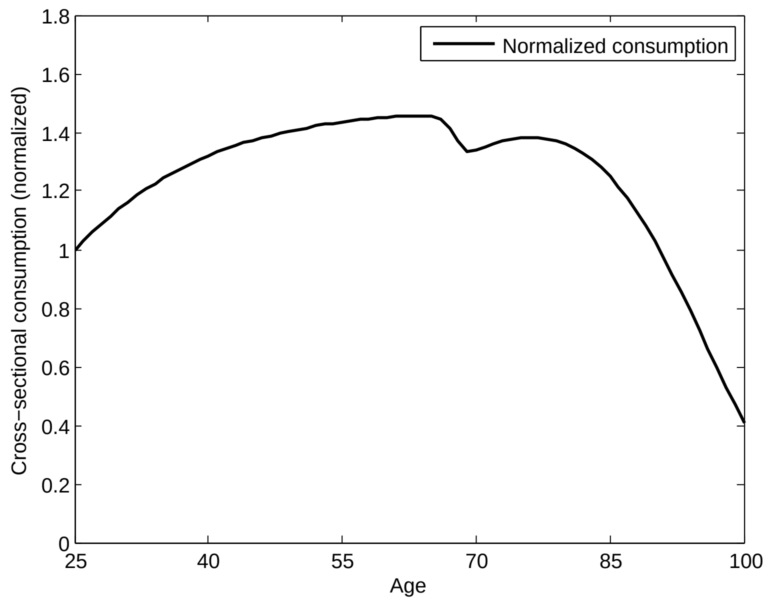

Figure 12: Cross-sectional mean of consumption (normalized by consumption at age 25) with $\sigma = 6$.

under $\sigma = 4$, the percentage declines in the Social Security benefit are somewhat larger. The actuarial projections (Case 1) overestimate the Social Security crisis by 24% on the average under $\sigma = 1$, compared to 34-37% under $\sigma = 4$.

Now let us consider the case of $\sigma = 6$. The unobservable parameter values for which the model matches the targets under $\sigma = 6$ are reported in Table 13, and model performance under these parameter values is reported in Table 14. The baseline normalized cross-sectional means consumption and labor hours in this case are reported in Figures 12 and 13 respectively.

With $\sigma = 6$, the drop in consumption at the date of retirement (about age 69) is larger

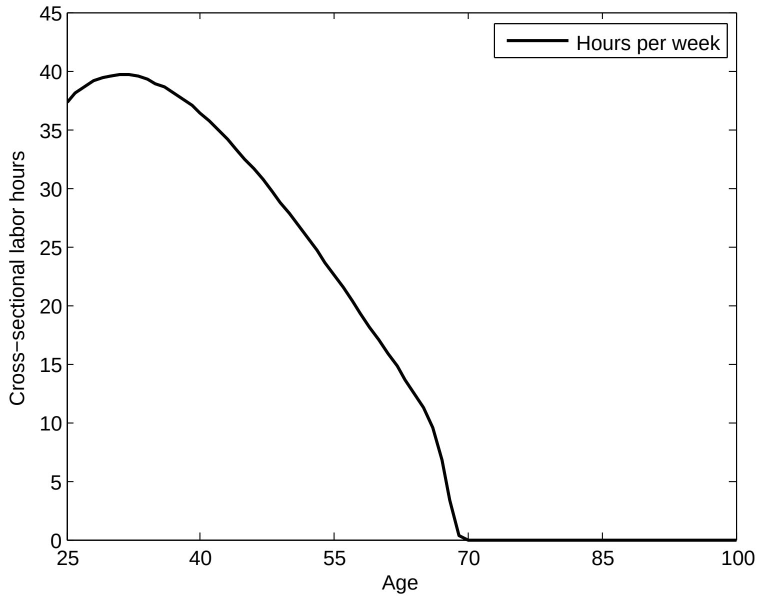

Figure 13: Cross-sectional mean of hours per week with $\sigma = 6$.

Projection	Case 1	Case 2	Case 2 relative to Case 1
A	-11.5%	-6.65%	57.8%
B	-24.5%	-14.4%	58.6%
C	-35.4%	-21.1%	59.6%

Table 15: Percentage changes in the Social Security benefit under Cases 1 and 2 with $\sigma = 6$.

than it is for $\sigma = 4$ (see Figure 12). Between age 65 and 68 consumption falls by about 8% under $\sigma = 6$, but only 4.3% under $\sigma = 4$. In this case, cross-sectional mean consumption peaks at age 64, with a ratio of peak-to-initial consumption of 1.46. The peak is only slightly smaller, and occurs a year earlier than under $\sigma = 4$, but is still significantly larger than the data. Also, the peak in labor hours shifts to slightly earlier in life under $\sigma = 6$ (see Figure 13), and retirement occurs slightly later.

Table 15 reports the effect on the budget-balancing Social Security benefit under $\sigma = 6$ for all the three demographic experiments. As before, for Case 2 the percentage changes are averaged across the different household types. The table shows that with $\sigma = 6$, the percentage decline in the budget-balancing Social Security benefit under Case 2 is only about 58-60% of that under Case 1. The increases in labor supply from the baseline under Case 2 are 4.2%, 9.9%, and 15.9% respectively, which are larger than those with $\sigma = 4$. Averaged across all ages, weekly labor supply increases from the baseline by roughly 38, 93, and 154 minutes, which are also larger than the increases with $\sigma = 4$ (see Figure 14).

The increases in cross-sectional mean asset holding are smaller under $\sigma = 6$: roughly 21%, 44%, and 64% respectively for the three experiments. However, the percentage increases in the aggregate capital stock from the baseline are actually larger under $\sigma = 6$: 11.3%, 27%, and 44%. This, once again, has to do with how the assets are distributed by

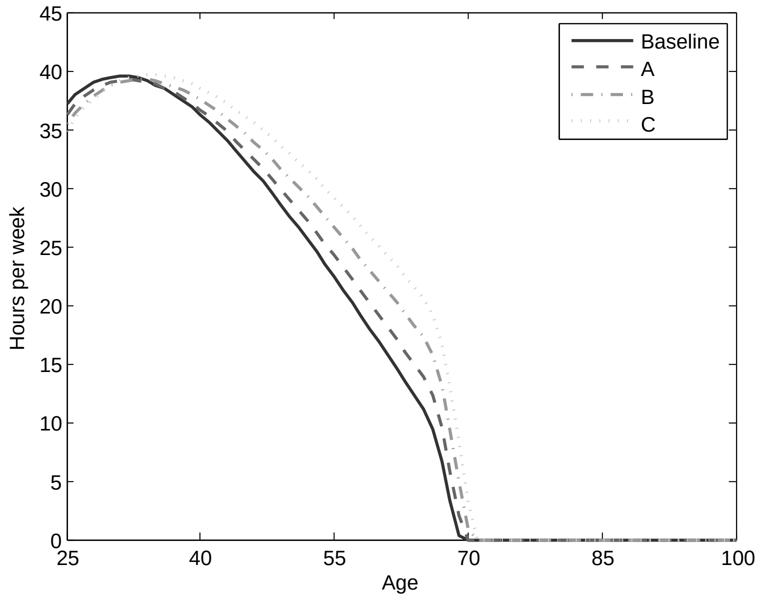

Figure 14: Cross-sectional means of labor hours per week, baseline and post-experiment, with $\sigma = 6$.

age. Old-age asset holdings are significantly larger under $\sigma = 6$, and because most of the improvements in survivorship occur at these ages, the smaller increases in the cross-sectional mean asset holding are more than compensated for.

With $\sigma = 6$, the equilibrium wage rate increases from the baseline by 2.3%, 5.2%, and 8.0% respectively. These are larger than what we find with $\sigma = 4$, and as a result the Social Security tax base increases by roughly 6.5%, 15.1%, and 23.9%. Because the improvements in longevity lead to larger increases in the tax base under $\sigma = 6$, we find that the actuarial projections overestimate the Social Security crisis by 40-42%, compared to 36-37% under $\sigma = 4$.

To summarize, I find that the general-equilibrium estimates of the Social Security crisis are always significantly smaller than the SSA's actuarial projections. The SSA's actuarial projections (Case 1) always overestimate the crisis, although the extent of overestimation is slightly larger for a higher $\sigma-$value, or a lower intertemporal elasticity.

9 Conclusions

The SSA's actuarial projections overestimate the Social Security crisis as they ignore the endogenous household-level and macroeconomic adjustments associated with improvements in longevity. I show that in general equilibrium, the budget-balancing decline in the Social Security benefit is only about 63% of what the actuarial estimates predict. There are two reasons behind this: households respond to a higher life expectancy by working more hours over the life-cycle and delaying retirement, which increases the aggregate labor supply, and also by saving more, which increases the capital stock and therefore the wage rate.

Collectively, these changes lead to a natural expansion in the Social Security tax base that the actuarial estimates completely overlook. I also find that the effect of labor supply is quantitatively more important than that of the wage rate: ignoring only the wage rate adjustment leads to a much smaller overestimation of the crisis. Finally, I find that this result is not sensitive to the values of the structural parameters used to calibrate the general-equilibrium model.

The mechanisms that I consider in this paper are a part of almost all standard macroeconomic models used to evaluate the effects of counterfactual policy experiments. My findings make a strong case in favor of including these mechanisms in the metrics that the SSA routinely uses to measure the health of Social Security in the U.S.

References

Arias, E. (2004). United States Life Tables, 2001. *National Vital Statistics Reports 52*(14).

Bullard, J. and J. Feigenbaum (2007). A Leisurely Reading of the Life-Cycle Consumption Data. *Journal of Monetary Economics 54*(8), 2305–2320.

Chen, K. and A. İmrohoroğlu (2012). Debt and the U.S. Economy. *Working Paper*.

Conesa, J. C. and C. Garriga (2008a). Optimal Fiscal Policy In The Design Of Social Security Reforms. *International Economic Review 49*(1), 291–318.

Conesa, J. C. and C. Garriga (2008b). Optimal Response to a Transitory Demographic Shock in Social Security Financing. In R. Fenge, G. de Ménil, and P. Pestieau (Eds.), *Pension Strategies in Europe and the United States*, pp. 98–113. Cambridge, MA: MIT Press.

De Nardi, M., S. İmrohoroğlu, and T. J. Sargent (1999). Projected U.S. Demographics and Social Security. *Review of Economic Dynamics 2*(3), 575–615.

Dushi, I. and A. Webb (2004). Household Annuitization Decisions: Simulations and Empirical Analysis. *Journal of Pension Economics and Finance 3*(2), 109–143.

Feigenbaum, J. (2008). Can mortality risk explain the consumption hump? *Journal of Macroeconomics 30*(3), 844 – 872.

Gourinchas, P.-O. and J. A. Parker (2002). Consumption over the Life Cycle. *Econometrica 70*(1), 47–89.

Guo, L., F. N. Caliendo, and R. Hosseini (2012). Social Security is NOT a Substitute for Annuities. *Working Paper*.

Hansen, G. D. (1993). The Cyclical and Secular Behaviour of the Labour Input: Comparing Efficiency Units and Hours Worked. *Journal of Applied Econometrics 8*(1), 71–80.

Hendricks, L. (2002). Intended and Accidental Bequests in a Life-cycle Economy. *Working Paper*.

Huang, H., S. İmrohoroğlu, and T. J. Sargent (1997). Two Computations to Fund Social Security. *Macroeconomic Dynamics 1*(1), 7–44.

Huggett, M. (1996). Wealth distribution in life-cycle economies. *Journal of Monetary Economics 38*(3), 469–494.

Huggett, M. and G. Ventura (1999). On the Distributional Effects of Social Security Reform. *Review of Economic Dynamics 2*(3), 498–531.

İmrohoroğlu, A., S. İmrohoroğlu, and D. H. Joines (1995). A Life Cycle Analysis of Social Security. *Economic Theory 6*(1), 83–114.

Jeske, K. (2003). Pension systems and aggregate shocks. *Federal Reserve Bank of Atlanta Economic Review* (Q1), 15–31.

Kitao, S. (2012). Sustainable Social Security: Four Options. *Working Paper*.

Ljungqvist, L. (2008). Lucas critique. In S. N. Durlauf and L. E. Blume (Eds.), *The New Palgrave Dictionary of Economics*. Basingstoke: Palgrave Macmillan.

Lockwood, L. M. (2012). Bequest motives and the annuity puzzle. *Review of Economic Dynamics 15*(2), 226–243.

Mitchell, O. S., J. M. Poterba, M. J. Warshawsky, and J. R. Brown (1999). New Evidence on the Money's Worth of Individual Annuities. *The American Economic Review 89*(5), 1299–1318.

Nishiyama, S. and K. Smetters (2005). Does Social Security Privatization Produce Efficiency Gains? Working Paper 11622, National Bureau of Economic Research.

Ortiz, J. A. (2009). Social Security and Retirement across OECD Countries. *Working Paper*.

Pashchenko, S. (2010). Accounting for non-annuitization. *Working Paper*.

Prescott, E. C., R. Rogerson, and J. Wallenius (2009). Lifetime aggregate labor supply with endogenous workweek length. *Review of Economic Dynamics 12*(1), 23–36.

Rogerson, R. and J. Wallenius (2009). Micro and macro elasticities in a life cycle model with taxes. *Journal of Economic Theory 144*(6), 2277–2292.

Shrestha, L. B. (2006). Life Expectancy in the United States. CRS Report for Congress RL32792, Congressional Research Service.

The Urban Institute (2002). Modeling Income in the Near Term: Revised Projections of Retirement Income Through 2020 for the 1931-1960 Birth Cohorts.

Turra, C. and O. S. Mitchell (2004). The Impact of Health Status and Out-of-Pocket Medical Expenditures on Annuity Valuation. *Michigan Retirement Research Center Research Paper WP 2004-086*.

Wallenius, J. (2009). Social Security and Cross-Country Hours: A General Equilibrium Analysis. *Working Paper*.