Annuity Markets and Capital Accumulation

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Abstract

We examine how the absence of annuities in financial markets affects capital accumulation in a two-period overlapping generations model. Our findings indicate that the effect on capital is ambiguous in general equilibrium, because there are two competing mechanisms at work. On the one hand, the absence of annuities increases the price of old-age consumption relative to the price of early-life consumption. This induces a substitution effect that reduces saving and capital, and an income effect that has the opposite effect as households want to consume less when young, causing them to save more. On the other hand, accidental bequests originate from the assets of the deceased under missing annuity markets. The bequest received in early life always has a positive income effect on saving, but the bequest received in old age, conditional on survival, is effectively a partial annuity with both substitution and income effects. We find that when the desire to smooth consumption is high, the income effects dominate, so the capital stock always increases when annuity markets are missing. However, when the desire to smooth consumption is low, the substitution effects dominate, and the capital stock decreases with missing annuity markets.

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1 Introduction

Since Heijdra et al. (2014) and Feigenbaum et al. (2013), it has been known that, contrary to intuition based on Yaari's (1965) partial equilibrium finding that households are better off investing in longevity annuities that pay a stream of income until death, there exist conditions under which households in general equilibrium can be better off if they do not have access to longevity annuities. One mechanism by which this result comes about is that the equilibrium capital stock can be larger in an economy without longevity annuities. Since, unlike household welfare, the capital stock and related macroeconomic variables such as aggregate output are observable, here we set out to identify the mechanisms by which annuities or the absence thereof affect the capital stock in a simple two-period overlapping generations model.

In a neoclassical framework, capital is accumulated through the aggregation of saving at the household level. From the perspective of the household, longevity annuities affect the lifetime budget constraint by increasing the effective return on saving. Some households that purchase annuities will not survive till the next period, and the value of their annuities will be distributed across the households that do survive, elevating the rate of return of annuities above the return on capital.[1] In a two-period model, this alters the price of consumption when old relative to the price of consumption when young. As Yaari (1965) shows, a rational household with no bequest motive will fully take advantage of annuities to lower the cost of consumption when old. But this change in relative prices induces both a substitution effect and an income effect.

As is often the case when both of these effects may be present, common intuition focuses only on the substitution effect, which is easier to comprehend. Households with access to annuities will respond to the cheaper cost of consumption in old age by consuming less and saving more when young. Often this is described in terms of households using annuities to obtain longevity insurance. In the aggregate, this substitution effect will cause the capital stock to increase in response to the opening of annuity markets.

What the preceding analysis ignores, however, is that the lower relative price of old-age consumption will also cause a contrary income effect. Assuming, as we do, that all income is earned while young, decreasing the price of consumption when old will pivot the budget line so as to expand the budget set. The household will be able to afford new bundles where it consumes more while young and, even though it saves less, also while old on account of the higher rate of return. Consequently, without considering specific preferences, the effect of this change in relative prices on aggregate saving is ambiguous. If the household's preference for consumption smoothing is high, the income effect will dominate, and the change in relative prices will decrease saving and the capital stock. On the other hand, if the preference for consumption smoothing is low, the substitution effect will dominate, and this change in relative prices will increase saving and the capital stock. With constant relative risk aversion (CRRA) preferences, the desire to smooth consumption depends on the elasticity of intertemporal substitution. If the wage and return on capital did not depend on the household's decisions (which, of course, they do in general equilibrium), the threshold value of the elasticity for which the income and substitution effects cancel out each other would be unity.

However, there is yet another effect of opening or closing annuities markets that must be considered in general equilibrium. What happens to the assets of a deceased household after death? The answer to this question is well known under perfect annuities markets (Yaari, 1965): the assets are redistributed to the survivors in the same cohort of annuity investors, resulting in a higher rate of return for those who survive to the following age. But something must also be done with the assets of the deceased in an economy where annuity markets are missing. Here we follow the norm in the literature and assume that the assets of the deceased are distributed uniformly across the surviving population as an accidental bequest. The bequest inherited while young constitutes a pure and positive income effect on households: the budget line shifts out, permitting households to consume more while young and old, and the latter can only happen with an increase in saving. Thus the bequest received while young increases saving regardless of preferences.[2]

The bequest inherited while old, on the other hand, is effectively another longevity annuity. One can only receive this latter bequest conditional on survival to old age, and the bequest is a partial redistribution of the assets of the households that do not survive to old age. As a result, for a household that survives to old

[1] More precisely, the rate of return on annuitized wealth at each age in Yaari's (1965) setup is the rate of return on capital plus the hazard rate of dying at that age, which follows from the assumption of perfectly competitive annuity providers with zero profits.

[2] This result does assume that the household views consumption at both ages as normal goods.

age, the effective return on saving is higher than the return on capital. This, as we already know, alters the relative price of old-age consumption and induces conflicting income and substitution effects. Comparing the income and substitution effects that derive from a pure longevity annuity as in Yaari (1965) to the income and substitution effects that derive from the accidental bequest, we show that the overall effect of missing annuity markets on the capital stock is ambiguous. We find that when the elasticity of intertemporal substitution is low, the income effects dominate, so the capital stock always increases when annuity markets are missing. However, when the intertemporal elasticity is high, the substitution effects dominate, and the capital stock decreases with missing annuity markets.

It is important to note here that with missing annuity markets, the income effect resulting from a lower relative price of old-age consumption always works in the same direction, increasing saving, while the corresponding substitution effect decreases saving. The accidental bequest received while young, on the other hand, always increases saving. We find that accounting for the income and substitutions effects of the partial longevity annuity from the accidental bequest pushes the threshold elasticity of intertemporal substitution to an even lower level. We show that this threshold elasticity is somewhere between unity and one half depending on the survival probability to old age, and for a typical calibration to the U.S., it is equal to 0.502 – much closer to the lower bound of the interval. Finally, when we account for the positive income effect from the accidental bequest received while young, this threshold elasticity increases to 0.826 under parameter values consistent with the U.S. economy, but is still lower than unity.[3]

This paper contributes to the literature on the economic implications of imperfections in insurance markets. Deaton and Paxson (1994) were the first to identify the importance of uninsurable income risk in explaining the evolution of consumption inequality with age. In a later study, Deaton and Paxson (1998) also identify health risk as being an important determinant of income risk. Since then, numerous studies have attempted to measure and identify the implications of income and health risks on consumption, saving, and labor supply. On the other hand, Davies (1989) and Hurd (1989) identify uninsurable mortality risk as being an important determinant of the old-age saving behavior. We complement this literature by demonstrating how the inability to insure against a specific type of risk – mortality risk – can affect overall capital accumulation in a general-equilibrium overlapping-generations model.

Finally, we have only mentioned longevity insurance obliquely in the preceding discussion. This is because longevity insurance, as it is usually understood, does not play any role in our model either with or without annuities. Longevity risk is the danger of outliving one's assets, which is impossible in a rational expectations framework. There is a maximum possible age (of two in our model) and households are fully informed of what that age is. This is not to say that longevity risk is something that policymakers or households should not be concerned about, but it can be meaningfully studied only in models with irrational expectations where the probability of living to any particular age is higher than what households believe the probability to be.

The rest of the paper is organized as follows: in Section 2, we present the model and derive the conditions needed to solve for the equilibrium capital stock with and without annuity markets. In Section 3 we demonstrate how the equilibrium capital stock under the two regimes depends on consumption smoothing, and ultimately the elasticity of intertemporal substition. We conclude in Section 4.

2 The Model

Consider a two-period economy in which households work when young (period 0) and are retired if they survive to old age (period 1). Let $0 < Q < 1$ be the probability of surviving till old age. Then, a household's objective is to maximize

$$U = u(c_0) + \beta Q u(c_1) \tag{1}$$

where c_0 and c_1 are the respective period consumptions, $\beta \geq 0$ is the discount factor, and $u(\cdot)$ is a strictly concave period utility function. Households maximize this lifetime expected utility function subject to the budget constraints, which depend on the state of annuity markets. We consider two alternative regimes: Regime A, when annuity markets exist and are frictionless (Yaari, 1965), and Regime B, when annuity markets are nonexistent. Output is produced using capital, labor, and a constant returns to scale production

[3] We use the average death rate between ages 60-70 from the U.S. Life Tables in Arias (2004) to calculate the probability of surviving to old age.

function with diminishing returns to capital. Factor markets are perfectly competitive, which implies that factor prices are equal to their marginal products, and the total quantity of labor is normalized to unity.

2.1 Regime A: Frictionless Annuity Markets

When annuity markets are frictionless (Yaari, 1965), the period 0 budget constraint is given by

$$c_0 + a_1 = w \tag{2}$$

where w is the wage, and the period 1 budget constraint is given by

$$c_1 = \frac{R}{Q} a_1, \tag{3}$$

where a_1 is the amount of annuities purchased in the young age, and R is the gross rate of return of capital. The return of a unit of annuity, however, is $\frac{R}{Q}$, because a fraction $1 - Q$ of households do not survive into retirement, and their annuitized wealth must be redistributed to the surviving households to ensure zero profits in the annuities market.[4]

The aggregate capital stock in this regime is given by

$$k = a_1. \tag{4}$$

Since labor is normalized to 1, we can denote the strictly concave production function by $f(k)$. Then the equilibrium wage rate is

$$w(k) = f(k) - kf'(k) \tag{5}$$

and the equilibrium (gross) rate of return of capital is

$$R(k) = f'(k) + 1 - \delta, \tag{6}$$

where $\delta \in [0, 1]$ is the depreciation rate.

Substituting in the budget constraints, the household's objective function in Regime A can therefore be written as

$$U(k) = u(w - k) + \beta Q u\left(\frac{R}{Q} k\right), \tag{7}$$

with the first-order condition

$$-u'(w - k) + \beta R u'\left(\frac{R}{Q} k\right) = 0,$$

which is nothing but the Euler equation

$$u'(c_0) = \beta R u'(c_1). \tag{8}$$

The equilibrium capital stock in Regime A (k_a^*) will therefore solve

$$u'(f(k_a^*) - k_a^* f'(k_a^*) - k_a^*) = \beta\left(f'(k_a^*) + 1 - \delta\right) u'\left(\frac{f'(k_a^*) + 1 - \delta}{Q} k_a^*\right). \tag{9}$$

2.2 Regime B: Missing Annuity Markets

When annuity markets are missing or nonexistent, we assume that the assets of the deceased are redistributed back to the surviving households in a lump-sum fashion as an accidental bequest B. In this regime, the period 0 budget constraint is given by

$$c_0 + b_1 = w + B, \tag{10}$$

[4] Note that because $Q < 1$, $\frac{R}{Q} > R$, so annuities offer a strictly larger return than capital. As a result, all wealth is annuitized in this regime.

and the period 1 budget constraint is given by

$$c_1 = Rb_1 + B, \tag{11}$$

where the accidental bequest B satisfies

$$(1+Q)B = (1-Q)Rb_1. \tag{12}$$

The aggregate capital stock in this regime is given by

$$k = b_1. \tag{13}$$

Substituting the budget constraints (10) and (11), the household's objective function in Regime B can therefore be written as

$$U(k) = u(w + B - k) + \beta Q u(Rk + B) \tag{14}$$

with the first-order condition

$$-u'(w + B - k) + \beta RQ u'(Rk + B) = 0,$$

which, once again, is nothing but the Euler equation

$$u'(c_0) = \beta RQ u'(c_1). \tag{15}$$

Combining Eqs. (12) and (13), we obtain the equilibrium bequest

$$B(k) = \frac{1-Q}{1+Q} Rk = \frac{1-Q}{1+Q} \left(f'(k) + 1 - \delta\right) k. \tag{16}$$

Solving for c_0 and c_1 as functions of the capital stock k and inserting these into (15), we find the equilibrium capital stock in Regime B, k_b^*, will solve

$$u'\left(f(k_b^*) - k_b^* f'(k_b^*) - k_b^* + \frac{1-Q}{1+Q}\left(f'(k_b^*) + 1 - \delta\right) k_b^*\right) = \beta\left(f'(k_b^*) + 1 - \delta\right) Q u'\left(\frac{2}{1+Q}\left(f'(k_b^*) + 1 - \delta\right) k_b^*\right) \tag{17}$$

To compare the equilibrium capital stock under frictionless annuity markets to that under missing annuity markets, we therefore need to compare conditions (9) and (17).

3 Results

Let us assume a Constant Relative Risk Aversion (CRRA) utility function

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma},$$

where $1/\gamma$ is the elasticity of intertemporal substitution. Under this assumption, conditions (9) and (17) simplify to

$$f(k_a^*) - k_a^* f'(k_a^*) - k_a^* = \frac{\left(\beta\left(f'(k_a^*) + 1 - \delta\right)\right)^{-1/\gamma}}{Q}\left(f'(k_a^*) + 1 - \delta\right) k_a^* \tag{18}$$

and

$$f(k_b^*) - k_b^* f'(k_b^*) - k_b^* = \frac{2\left(\beta\left(f'(k_b^*) + 1 - \delta\right) Q\right)^{-1/\gamma} - 1 + Q}{1+Q}\left(f'(k_b^*) + 1 - \delta\right) k_b^*. \tag{19}$$

Let us now define

$$g(k) = \frac{f(k) - kf'(k) - k}{(f'(k) + 1 - \delta)k}. \tag{20}$$

Then, we can write the equilibrium conditions (18) and (19) as

$$g(k_i^*) = m_i \times \left(\beta R(k_i^*)\right)^{-1/\gamma} + h_i \tag{21}$$

where $i = a, b$. Thus,

$$m_a = \frac{1}{Q} \quad \text{and} \quad h_a = 0, \tag{22}$$

and

$$m_b = \frac{2}{1+Q} Q^{-1/\gamma} \quad \text{and} \quad h_b = -\frac{1-Q}{1+Q}. \tag{23}$$

We can interpret m_a as delivering the income and substitution effects that result from the effect of annuities markets on the relative prices of early-life and old-age consumption. Likewise, m_b delivers the income and substitution effects that result from the partial annuitization of assets through the bequest mechanism. Finally, h_b represents the pure income effect coming from an accidental bequest earned while young. Notice that if $Q = 1$, so that there is no mortality risk, then $m_a = m_b = 1$ and $h_a = h_b = 0$. In that special case, there is no difference between the two regimes.

To understand the nature of this equilibrium condition, we first state the following:

Lemma 1 *With a Cobb-Douglas production function $f(k) = k^\alpha$ and $0 < \alpha < 1$, $g(\cdot)$ is strictly decreasing in k.*

Proof.

$$g(k) = \frac{f(k) - kf'(k) - k}{(f'(k) + 1 - \delta)k} = \frac{k^\alpha - k\alpha k^{\alpha-1} - k}{(\alpha k^{\alpha-1} + 1 - \delta)k} = \frac{(1-\alpha)k^\alpha - k}{\alpha k^\alpha + (1-\delta)k}$$

Therefore,

$$\begin{aligned} g'(k) &= \frac{[\alpha(1-\alpha)k^{\alpha-1} - 1][\alpha k^\alpha + (1-\delta)k] - [\alpha^2 k^{\alpha-1} + 1 - \delta][(1-\alpha)k^a - k]}{[\alpha k^\alpha + (1-\delta)k]^2} \\ &= \frac{\alpha^2(1-\alpha)k^{2\alpha-1} - \alpha k^\alpha + \alpha(1-\alpha)(1-\delta)k^\alpha - (1-\delta)k - \alpha^2(1-\alpha)k^{2\alpha-1}}{[\alpha k^\alpha + (1-\delta)k]^2} \\ &\quad - \frac{(1-\delta)(1-\alpha)k^\alpha + \alpha^2 k^\alpha + (1-\delta)k}{[\alpha k^\alpha + (1-\delta)k]^2} \\ &= \frac{-\alpha(1-\alpha)k^\alpha - (1-\alpha)^2(1-\delta)k^\alpha}{[\alpha k^\alpha + (1-\delta)k]^2} < 0 \end{aligned}$$

■

Lemma 2 *With a Cobb-Douglas production function $f(k) = k^\alpha$ and $0 < \alpha < 1$, $R(.)^{-1/\gamma}$ is strictly increasing in k.*

Proof.

$$\begin{aligned} R(k)^{-1/\gamma} &= \left(\alpha k^{\alpha-1} + 1 - \delta\right)^{-1/\gamma} \\ \frac{d}{dk}\left[R(k)^{-1/\gamma}\right] &= -\frac{\left(\alpha k^{\alpha-1} + 1 - \delta\right)^{-1/\gamma-1}}{\gamma} \quad \alpha(\alpha-1)k^{\alpha-2} > 0 \end{aligned}$$

■

With Lemmas 1 and 2, we can now state the following proposition:

Proposition 3 *There is a unique equilibrium capital stock k^* in both regimes.*

Proof. From equation (21), the equilibrium condition for capital stock is

$$g(k_i^*) = m_i \times (\beta R(k_i^*))^{-1/\gamma} + h_i$$

From equations (22) and (23), h_i is independent of k. Also, from Lemmas 1 and 2, the LHS of the equilibrium condition is strictly decreasing and the RHS is strictly increasing in k. Therefore, there is a unique capital stock k^* in both regimes. ■

Now, let us define γ^* as the value of the inverse of intertemporal elasticity for which $m_a = m_b$ from equations (22) and (23), or when the income and substitution effects from a change in the relative price of old-age consumption originating from the annuity markets exactly cancel out the income and substitution effects that originate from the partial annuitization of assets through the accidental bequests. This critical value is given by

$$\gamma^* = \frac{\ln Q}{\ln\left(\frac{2Q}{1+Q}\right)}. \tag{24}$$

Because

$$\frac{\partial}{\partial\gamma}\left(\frac{m_b}{m_a}\right) = -\frac{1}{\gamma}\frac{2Q^{-\frac{1+\gamma}{\gamma}}}{1+Q} < 0,$$

if $\gamma \geq \gamma^*$, then $m_a \geq m_b$. In the limit as $Q \to 0$,

$$\lim_{Q\to 0}\frac{\ln Q}{\ln\left(\frac{2Q}{1+Q}\right)} = \lim_{Q\to 0}\frac{\ln Q}{\ln(2Q)} = \lim_{Q\to 0}\frac{\frac{1}{Q}}{\frac{1}{Q}} = 1.$$

In the limit as $Q \to 1$,

$$\lim_{Q\to 1}\frac{\ln Q}{\ln\left(\frac{2Q}{1+Q}\right)} = \lim_{Q\to 1}\frac{\ln Q}{\ln 2 + \ln Q - \ln(1+Q)} = \lim_{Q\to 1}\frac{\frac{1}{Q}}{\frac{1}{Q} - \frac{1}{1+Q}} = \lim_{Q\to 1}\frac{1}{1-\frac{Q}{1+Q}} = \frac{1}{1-\frac{1}{2}} = 2.$$

Therefore, the bound on γ^* is always between unity and 2, or the bound on the elasticity of intertemporal substitution $(1/\gamma^*)$ is always between unity and one half, depending on Q. For a typical calibration to the U.S., the average death rate between ages 60-70 is about 1.4% (Arias, 2004), which yields a threshold elasticity value of 0.502.

Meanwhile, also from equations (22) and (23), we have $h_a > h_b$. Therefore, we can conclude that if $\gamma \geq \gamma^*$, then

$$m_a \times (\beta R(k))^{-1/\gamma} + h_a \quad \geq \quad m_b \times (\beta R(k))^{-1/\gamma} + h_b,$$

which implies from Lemma 1 that $k_a^* \leq k_b^*$ or that missing annuity markets increases capital stock. On the other hand, if $\gamma < \gamma^*$, then

$$m_a \times (\beta R(k))^{-1/\gamma} + h_a \quad \lesseqgtr \quad m_b \times (\beta R(k))^{-1/\gamma} + h_b,$$

which implies that $k_a^* \lesseqgtr k_b^*$ or that the effect on capital stock is ambiguous. We formalize this result in the following proposition:

Proposition 4 *If the elasticity of intermporal substitution $(1/\gamma)$ is sufficiently low, both the changes in m and h cause k^* to increase when annuity markets are missing. When the elasticity of intertemporal substitution is high, the changes in m and h work in opposite directions and the effect on k^* is ambiguous.*

Proof. First, note that if $\gamma \geq \gamma^*$, then $m_a \geq m_b$ and vice versa, and $h_a > h_b$ regardless of γ. Moreover,

$$g(k^*) = m(\beta R(k^*))^{-1/\gamma} + h$$

$$g'(k^*)dk^* = dm(\beta R(k^*))^{-1/\gamma} - \frac{1}{\gamma}m\beta^{-1/\gamma}R(k^*)^{-\frac{1+\gamma}{\gamma}}R'(k^*)dk^* + dh$$

$$\left(g'(k^*) + \frac{1}{\gamma}m\beta^{-1/\gamma}R(k^*)^{-\frac{1+\gamma}{\gamma}}R'(k^*)\right)dk^* = dm(\beta R(k^*))^{-1/\gamma} + dh$$

Therefore

$$\frac{dk^*}{dm} = \frac{(\beta R(k^*))^{-1/\gamma}}{g'(k^*) + \frac{1}{\gamma}m\beta^{-1/\gamma}R(k^*)^{-\frac{1+\gamma}{\gamma}}R'(k^*)} < 0$$

$$\frac{dk^*}{dh} = \frac{1}{g'(k^*) + \frac{1}{\gamma}m\beta^{-1/\gamma}R(k^*)^{-\frac{1+\gamma}{\gamma}}R'(k^*)} < 0$$

Thus, if $\gamma \geq \gamma^*$, then $k_a^* \leq k_b^*$, and if $\gamma < \gamma^*$, then $k_a^* \lesseqgtr k_b^*$. ■

The intuition behind this result is as follows. As noted earlier, m_a delivers the income and substitution effects from a change in the relative price of old-age consumption originating from the annuity markets, and m_b delivers the income and substitution effects that originate from the partial annuitization of assets through the accidental bequests. The change in m when annuity markets are missing, therefore, reflects the difference in the respective magnitudes of these effects, best understood through the differing consumption smoothing behavior across the two regimes. When γ is high and the elasticity of intertemporal substitution is low, a household's desire to smooth consumption is strong. As a result, the income effects dominate, and saving and capital increases when annuity markets are missing. On the other hand, when γ is low and the elasticity of intertemporal substitution is high, there is very little desire to smooth consumption. As a result, the substitution effects dominate, and saving and capital may well decrease with missing annuity markets.

Note that h_b reflects the pure income effect from the accidental bequest received while young, which is effectively a transfer of income from old-age to early life. This transfer always encourages saving due to the life cycle motive, regardless of the preferences. Let us denote as γ^{**} the inverse of the intertemporal elasticity for which $k_a^* = k_b^*$, or that for which the income and substitution effects from the change in the relative price of old-age consumption originating from the annuity markets exactly cancel out the income and substitution effects that originate from the accidental bequest, including the positive income effect from the bequest inherited in early life. Suppose a period corresponds to thirty years. Then, with a two-period discount factor of $\beta = 0.294 = 0.96^{30}$, a Cobb-Douglas production function with capital's share of 33%, and a depreciation rate of $\delta = 1$, which is not so unreasonable if a period is thirty years, we find that $\gamma^{**} = 1.21$, or that the threshold intertemporal elasticity is $1/\gamma^{**} = 0.826$ in general equilibrium. Therefore, accounting for the positive income effect from the bequest inherited in early life, missing annuity markets lead to an increase in the capital stock even when the elasticity of intertemporal substitution is somewhat higher. Thus, the desirability of consumption smoothing, and therefore the relative strengths of the income and substitution effects under the two regimes, holds the key to understanding the overall effect of missing annuity markets on the capital stock.

To summarize, we find that the overall effect of missing annuity markets on the capital stock is ambiguous. This is because there are two competing mechanisms to account for: the income and substitution effects from a change in the relative price of old-age consumption originating from the annuity markets, and the income and substitution effects that originate from the accidental bequests from the assets of the deceased. When consumption smoothing is highly desirable, the income effects dominate and the capital stock increases under missing annuity markets, but when the desire for consumption smoothing is low, the substitution effects dominate and the capital stock decreases when annuity markets are missing.

4 Conclusions

Contrary to intuition based on Yaari's(1965) partial equilibrium finding that households are better off investing in longevity annuities, in general equilibrium households can be better off if they do not have access to longevity annuities. This is because the equilibrium capital stock can be larger in an economy without longevity annuities. In this paper, we investigate the precise effect of missing annuity markets on capital stock using a two–period overlapping–generations model.

Our findings indicate that the overall effect of missing annuity markets on capital stock is ambiguous, because there are two competing mechanisms at work. On the one hand, the absence of annuities increases the price of old-age consumption relative to the price of early-life consumption. This induces a substitution effect that reduces saving and capital, and an income effect that has the opposite effect as households want to consume less when young and save more. On the other hand, accidental bequests originate from the assets of the deceased under missing annuity markets. The bequest received in early life always has a positive income effect on saving, but the bequest received in old age, conditional on survival, is effectively a partial annuity with both substitution and income effects. We find that when the elasticity of intertemporal substitution is low or the desire to smooth consumption is high, the income effects dominate, so the capital stock always increases when annuity markets are missing. However, when the intertemporal elasticity is high or the desire to smooth consumption is low, the substitution effects dominate, and the capital stock decreases with missing annuity markets.

Finally, it is worth noting that the key mechanisms that we have considered here have also been found to be relevant in the area of optimal capital taxation. As Bernheim (2001) shows, in a two-period life cycle model, the interest elasticity of saving can be positive or negative, so saving can either rise or fall in response to an increase in the after-tax rate of return. This is because an increase in the after-tax rate of return amounts to an uncompensated reduction in the price of old-age consumption. As a result, the associated substitution effect shifts consumption towards the future (thereby increasing saving), while the associated income effect is usually assumed to increase consumption in both periods (thereby reducing saving). In our framework, missing annuity markets effectively lead to an uncompensated *increase* in the price of old-age consumption, and therefore have the exact opposite income and substitution effects. The only additional mechanism in our framework is the pure income effect from the accidental bequest in early life, which always increases saving.

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